Final Exam

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st -1 $t-t_0$ e^{-st_0} $\delta(t) \to 1$ e^{-s}

1. Consider a continuous-time LTI system with input $x(t)=e^{-t}u(t)-e^{-1}e^{-(t-1)}u(t-1)$ and impulse response $h(t)=\delta(t)+e^{-1}\delta(t-1)+e^{-2}\delta(t-2)+e^{-3}\delta(t-3)+\cdots$.
   (a) Determine the Laplace transforms of $x(t)$ and $h(t)$. (6%)
   (b) Determine the output $y(t)$ using the results of (a). (6%) -1 $-1-s<0$ $n-n_0$ z^{-n_0} e^{-} $s>-1$ $t-t_0$ e^{-st_0}

2. Consider the following Laplace transform pair:

$$x(t) \xleftrightarrow{\mathcal{L}} \frac{s}{(s+2)^2},\ \operatorname{Re}\{s\}>-2.$$

   (a) Determine the time-domain signal $x(t)$. (5%) $s=-2+j$
   (b) Determine the Laplace transform of $\int_0^t x(3\tau)d\tau$. (5%) $s^2+4s+4=-1$

3. Consider the following five facts about a real signal $x(t)$ with Laplace transform $X(s)$: e^{st}
   1) $X(s)$ has exactly two poles.
   2) $X(s)$ has no zeros in the finite s-plane.
   3) $X(s)$ has a pole at $s=-2+j$.
   4) $e^{3t}x(t)$ is not absolutely integrable.
   5) $X(0)=2$.

   Determine $X(s)$ and specify its region of convergence. (10%) $s^2+4s+4+5t$

4. Consider a continuous-time LTI system for which the input $x(t)$ and output $y(t)$ are related by the following differential equation:

$$\frac{d^2y(t)}{dt^2}-\frac{dy(t)}{dt}-6y(t)=x(t).$$

$(1-\frac{1}{3}z^{-1})^{-1}$ $-(1-\frac{1}{3}z^{-1})^{-2}(\frac{-1}{3})(-1)z^{-2}$ $(-1)(-z)-3z^{-2}$

   (a) Determine the system function and sketch its pole-zero diagram. (4%)
   (b) Indicate all possible regions of convergence for the system function in (a) and determine the corresponding impulse responses. What are the stability and/or causality properties for each of the possible cases? (8%)

$1-az^{-1}$ $(-1)\frac{-(\frac{1}{3})(-1)z^{-2}}{\frac{1}{3}z^{-2}}$

5. Let $B(s)$ be the system function of a causal and stable Butterworth filter of order 2 and

$$B(s)B(-s)=\frac{1}{1+(s/(j\cdot 3))^6}.$$

$(1-3z^{-1})^{-1}$ $-(1-3z^{-1})^{-2}(-3)(-1)z^{-2}$ $(1)\ 1-az^{-1}$ $nx[n]$ $-z\frac{dX}{dz}$

   (a) Sketch the pole-zero diagram of $B(s)B(-s)$. (4%)
   (b) Determine the system function $B(s)$. (5%)
   (c) Plot the frequency response of the filter roughly and indicate the 3-dB frequency. (3%)

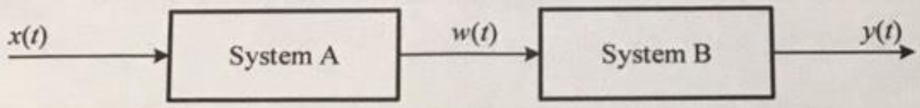
6. Determine the z-transform or the inverse z-transform for each of the following signals:
   (a) $n(1/3)^{|n|}$. (5%) (b) $X(z)=(z^{-2}+2z^{-1}+2)/(z^{-1}+1)$, $|z|>1$. (5%)

7. Consider a causal discrete-time LTI system with input $x[n]=(-1/5)^n u[n]-(1/3)^n u[-n-1]$ and output $y[n]=(-1/5)^n u[n]$.

   (a) Determine the z-transforms of $x[n]$ and $y[n]$. (6%)

   (b) Determine the corresponding impulse response. (6%)

8. Consider the following cascade interconnection of two continuous-time LTI systems:



   Also assume that the following three facts are given:

   - System A is causal with impulse response $h(t)=e^{-2t}u(t)$.
   - System B is causal and is characterized by the differential equation

$$\frac{dy(t)}{dt}+y(t)=\frac{dw(t)}{dt}+\alpha w(t).$$

   - The output is $y(t)=0$ when the input is $x(t)=e^{2t}$. $H(z) \cdot$ $H(s) \cdot e^{st}$

   (a) Find the system function $H(s)=Y(s)/X(s)$. Sketch the pole-zero diagram and indicate the corresponding region of convergence. (8%)

   (b) Determine the differential equation relating $y(t)$ and $x(t)$. (4%)

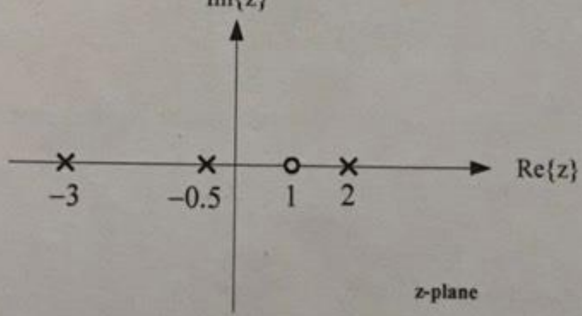
$\frac{1}{1+\frac{1}{3}z^{-1}}$

9. Consider the following difference equation:

$$y[n]+3y[n-1]=x[n],\ \ x[n]=(1/3)^n u[n] \text{ and } y[-1]=-1.$$

   Determine the zero-input and zero-state responses by using the unilateral z-transform.(10%) $\frac{>1}{7}$

10. Consider a discrete-time LTI system with system function $H_1(z)$ whose pole-zero diagram is shown as follows:



$a+b=1$

$-\frac{1}{3}a+3b=0$

$a-9b=0$

$a+3b=0$

   (a) How many two-sided impulse responses can be associated with this pole-zero diagram? Indicate the corresponding regions of convergence. (5%)

   (b) Consider a cascade interconnection of two systems $H_1(z)$ and $H_2(z)$. Determine a possible solution of $H_2(z)$ such that the overall system is causal and stable. (5%)