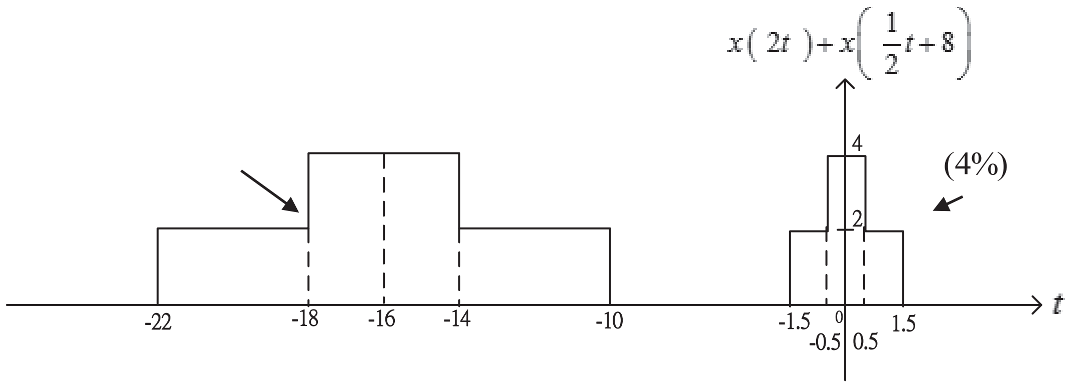
2018

1. Sketch the signal $x(2t)+x(\frac{1}{2}t+8)$.(10%)



2. (15%)

(1) $y(t)=(\cos(\pi t))x(t)$

(i) memoryless: 輸出只與當時的輸入有關

(ii) causal

(iii) stable: $|y(t)|=|\cos(\pi t)x(t)|\le M_x$

(iv) time-varying: $y(t)=H\{x(t)\}=\cos(\pi t)x(t)$
$H\{x(t-t_0)\}=\cos(\pi t)x(t-t_0)\neq y(t-t_0)$

(v) linear: $ay_1(t)+by_2(t)=H\{ax_1(t)+bx_2(t)\}$

(2) $y[n]=x[n^2]10$

(i) memory: 輸出與未來輸入有關

(ii) non-causal: 輸出與未來輸入有關

(iii) stable: $|y[n]|=|x[n^2]|\le M_x$

(iv) time-varying: $y[n]=H\{x[n]\}=x[n^2]$
$H\{x[n-1]\}=x[n^2-1]\neq y[n-1]$

(v) linear: $ay_1[n]+by_2[n]=H\{ax_1[n]+bx_2[n]\}$

(3) $y[n] = x[n]\sum_{k=0}^{\infty}\delta[n-k] \Rightarrow \begin{array}{ll} y[n]=0 & \text{for } n<0 \\ y[n]=x[n] & \text{for } n\geq 0 \end{array}$

(i) memoryless: 輸出只與當時的輸入有關

(ii) causal

(iii) stable

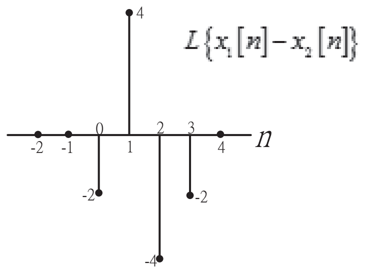
(iv) time-varying

(v) linear: $ay_1[n]+by_2[n]=H\{ax_1[n]+bx_2[n]\}$

3. (15%)

(1)

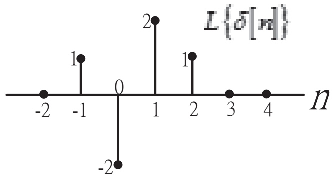
$$L\{x_1[n]-x_2[n]\}=y_1[n]-y_2[n]$$



(2)

$$\delta[n]=\frac{1}{2}x_1[n]-\frac{1}{2}x_2[n]+x_3[n]$$

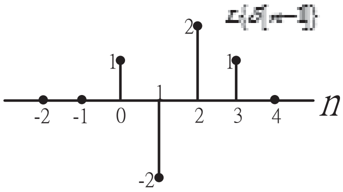
$$L\{\delta[n]\}=\frac{1}{2}y_1[n]-\frac{1}{2}y_2[n]+y_3[n]$$



(3)

$$\delta[n-1] = -\frac{1}{2}\left(x_1[n] - x_2[n]\right)$$

$$L\{\delta[n-1]\} = -\frac{1}{2}\left(y_1[n] - y_2[n]\right)$$



If the input $\delta[n]$ delay 1 unit, the output $L\{\delta[n]\}$ also delay 1 unit.

The system is time-invariant.

4.

$$y[n] - y[n-1] + \frac{1}{4}y[n-2] = \frac{1}{2}x[n],\ y[-1] = 1, y[-2] = 0.$$

$$\Rightarrow y^{(h)}[n] = c_1(\frac{1}{2})^n + c_2 n(\frac{1}{2})^n$$

(1) (7%)

$$\because x[n] = u[n] \therefore\ y^{(p)}[n] = A$$

$$\because y^{(p)}[n] - y^{(p)}[n-1] + \frac{1}{4}y^{(p)}[n-2] = \frac{1}{2}u[n]$$

$$\therefore A = 2 \Rightarrow y[n] = c_1(\frac{1}{2})^n + c_2 n(\frac{1}{2})^n + 2u[n].$$

$$\because y[0] = \frac{3}{2},\ y[1] = \frac{7}{4} \therefore c_1 = \frac{-1}{2},\ c_2 = 0.$$

$$\Rightarrow y[n] = \frac{-1}{2}(\frac{1}{2})^n + 2u[n].$$

(2) (8%)

$$y^{(n)}[n] = c_1(\frac{1}{2})^n + c_2 n(\frac{1}{2})^n,\ y[-1] = 1, y[-2] = 0 \Rightarrow$$

$$c_1 = 1,\ c_2 = \frac{1}{2} \Rightarrow y^{(n)}[n] = (\frac{1}{2})^n + \frac{1}{2}n(\frac{1}{2})^n.$$

$$y^{(f)}[n] = c_1(\frac{1}{2})^n + c_2 n(\frac{1}{2})^n + 2u[n],\ y[-1] = y[-2] = 0 \Rightarrow$$

$$y[0] = \frac{1}{2},\ y[1] = 1 \Rightarrow c_1 = \frac{-3}{2},\ c_2 = \frac{-1}{2} \Rightarrow$$

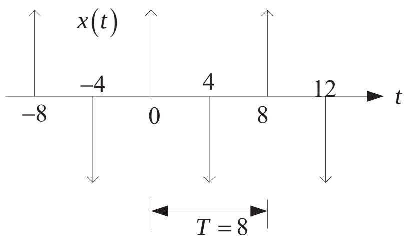
$$y^{(f)}[n] = \frac{-3}{2}(\frac{1}{2})^n + \frac{-1}{2}n(\frac{1}{2})^n + 2u[n].$$

5.

(10%)

(1)

Periodic, $T = 8$.



(2)

Periodic, $T_0 = \frac{2\pi}{10}$

(3)

Periodic, N=8

6.

(10%)

$$y(t) = 0, t < 0.5$$

$$2\int_{-0.5}^{t-1} \cos(\pi\tau)\, d\tau = \frac{2}{\pi}\left\{\sin\left[\pi(t-1)\right] + 1\right\}, 0.5 \le t < 2.5$$

$$2\int_{-3+t}^{1.5} \cos(\pi\tau)\, d\tau = \frac{2}{\pi}\left\{-\sin\left[\pi(t-3)\right] - 1\right\}, 2.5 \le t < 4.5$$

$$y(t) = 0, t > 4.5$$

7.

Homogeneous solution: $r^2 - 5r + 6 = 0 \Rightarrow r = 2, r = 3 \Rightarrow y^{(h)} = c_1 e^{2t} + c_2 e^{3t}$

(1) (6%)

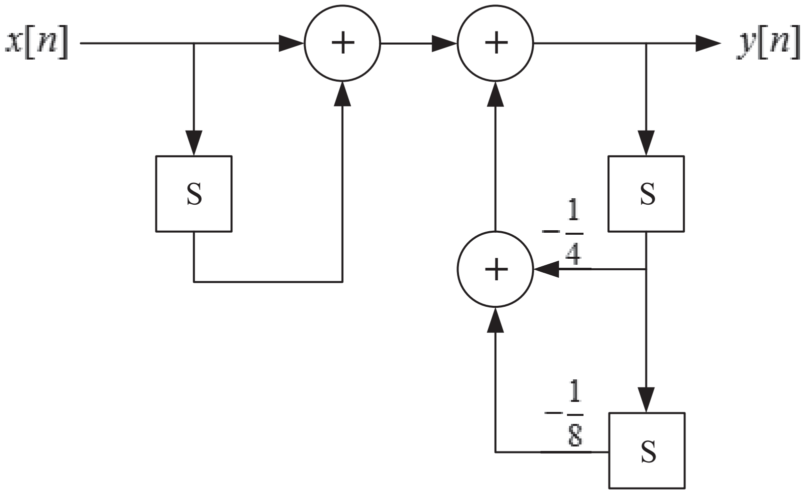
$$y_p(t) = A\cos(3t) + B\sin(3t),$$
$$y_p^{'}(t) = -3A\sin(3t) + 3B\cos(3t),$$
$$y_p^{"}(t) = -9A\cos(3t) - 9B\sin(3t),$$
$$A = \frac{5}{39}, \mathrm{B} = \frac{-1}{39}$$
$$\Rightarrow \therefore y_p(t) = \frac{-1}{39}\sin(3t) + \frac{5}{39}\cos(3t).$$

(2) (7%)

$$y_p(t) = Ate^{3t} + Bte^{2t},$$
$$y_p^{'}(t) = Ae^{3t} + 3Ate^{3t} + Be^{2t} + 2Bte^{2t},$$
$$y_p^{"}(t) = 6Ae^{3t} + 9Ate^{-3t} + 2Be^{2t} + 4Bte^{2t},$$
$$A = 2, B = -2$$
$$\Rightarrow \therefore y_p(t) = 2te^{3t} - 2te^{2t}.$$

8. (12%)

(1)

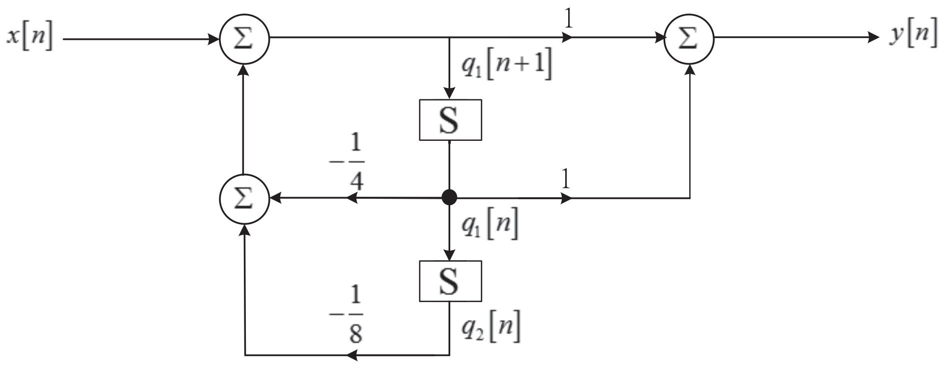


(I) $y[n]+\frac{1}{4}y[n-1]+\frac{1}{8}y[n-2]=x[n]+x[n-1]$

or

(II) $y[n]=x[n]+x[n-1]-\frac{1}{4}y[n-1]-\frac{1}{8}y[n-2]$

(2)



$$q_1[n+1]=-\frac{1}{4}q_1[n]-\frac{1}{8}q_2[n]+x[n]$$

$$q_2[n+1]=q_1[n]$$

$$y[n]=\frac{3}{4}q_1[n]-\frac{1}{8}q_2[n]+x[n]$$

$$\mathbf{A}=\begin{bmatrix}-\frac{1}{4} & -\frac{1}{8}\\ 1 & 0\end{bmatrix},\quad \mathbf{b}=\begin{bmatrix}1\\0\end{bmatrix},\quad \mathbf{c}=\left[\frac{3}{4},-\frac{1}{8}\right],\quad D=1$$

9.(10%)

(a)

$$13-7+1=7$$

(b)

$$
\begin{array}{r l}
 & 1\ 2\ 2\ 0\ -1\ 0\ 1 \\
\hline
1\ 1\ 2\ 3\ 1\ 2\ 4 \,) & 1\ 3\ 6\ 9\ \ 10\ 9\ 9\ 10\ 9\ 1\ -3\ 2\ 4 \\
 & 1\ 1\ 2\ 3\ \ 1\ \ 2\ 4 \\
\hline
 & \quad 2\ 4\ 6\ \ 9\ \ 7\ 5\ 10 \\
 & \quad 2\ 2\ 4\ \ 6\ \ 2\ 4\ \ 8 \\
\hline
 & \qquad 2\ 2\ \ 3\ \ 5\ 1\ \ 2\ 9 \\
 & \qquad 2\ 2\ \ 4\ \ 6\ 2\ \ 4\ 8 \\
\hline
 & \qquad\qquad -1-1-1-2\ \ 1\ \ 1\ -3 \\
 & \qquad\qquad -1-1-2-3-1-2-4 \\
\hline
 & \qquad\qquad\qquad 1\ \ 1\ \ 2\ \ 3\ \ 1\ 2\ 4 \\
 & \qquad\qquad\qquad 1\ \ 1\ \ 2\ \ 3\ \ 1\ 2\ 4 \\
\hline
 & \qquad\qquad\qquad\qquad\qquad\quad 0
\end{array}
$$

$$h[n]=\delta[n]+2\delta[n-1]+2\delta[n-2]-\delta[n-4]+\delta[n-6]$$