Midterm Exam II

Dec. 13, 2018

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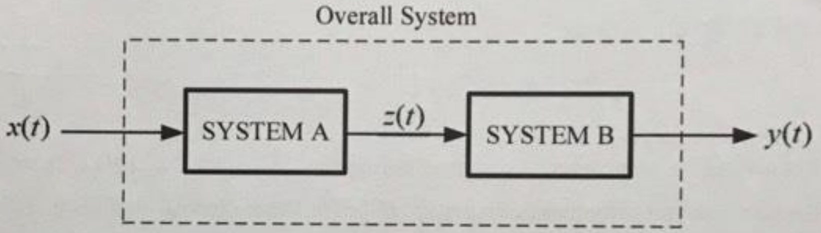
1. Consider a discrete-time linear time-invariant (LTI) system with frequency response

$$H(e^{j\Omega}) = \frac{1-e^{-j2\Omega}}{1+\frac{1}{2}e^{-j4\Omega}}, \quad -\pi \le \Omega \le \pi .$$

(a) Determine the output $y[n]$ when the input is $x[n] = \sin(\pi n/4) + \cos(\pi n/2)$. (5%)

(b) Determine the output $y[n]$ when the input is $x[n] = \sum_{k=-\infty}^{\infty} \delta[n-4k]$. (8%)

2. Consider the following causal system:



The input-output relation for SYSTEM A is characterized by the differential equation

$$\frac{dz(t)}{dt} + 5z(t) = \frac{dx(t)}{dt} + 3x(t)$$

$\frac{2\pi}{8}$

where the impulse response of SYSTEM B is given by

$$h_B(t) = \frac{1}{2}e^{-t}u(t) + \frac{1}{2}e^{-3t}u(t).$$

$\frac{3\pi}{4}$

(a) Determine the frequency response $H_A(j\omega)$ and the impulse response of SYSTEM A. (5%)

2π

(b) Determine the frequency response $H_B(j\omega)$ of SYSTEM B. (3%)

(c) Determine the frequency response of the overall system and the differential equation relating $x(t)$ and $y(t)$. (5%)

$\frac{3}{8}$

$8 \sum a_m b_{k-m}$

3. Consider the following three discrete-time signals with a fundamental period of ~~4~~ 8:

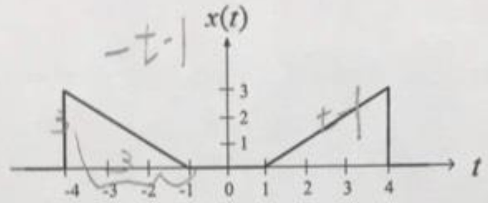
$$x[n] = \cos[(3\pi n/4) + (2\pi/3)],\ y[n] = 1 + \sin(\pi n/4),\ z[n] = x[n]y[n]$$

(a) Determine the Fourier series coefficients of $x[n]$ (4%)

(b) Determine the Fourier series coefficients of $y[n]$ (4%)

(c) Use the results of (a) and (b) to determine the Fourier series coefficients of $z[n]$. (5%)

4. Evaluate the quantities for the following signal:



(a) $\tan^{-1}\left\{\dfrac{\mathrm{Im}(X(j\omega))}{\mathrm{Re}(X(j\omega))}\right\}$. (3%) (b) $\int_{-\infty}^{\infty}|X(j\omega)|^2\,d\omega$. (4%)

(c) $\int_{-\infty}^{\infty}X(j\omega)e^{j2\omega}d\omega$. (4%)

$\tau = -t$ $t = -\tau$ $\int_4^1 t^2 - 2t + 1$ $\int_1^4 -t^2+2t-1$ $(t+1)^2$ $(t-1)^2$

5. An ideal low-pass filter (LPF) with zero delay has impulse response $h_{\mathrm{LP}}[n]$ and frequency response given by

$$H_{\mathrm{LP}}(e^{j\Omega}) = \begin{cases} 1, & |\Omega| < 0.2\pi, \\ 0, & 0.2\pi \le |\Omega| \le \pi. \end{cases}$$

(a) Let a filter be with impulse response defined by $h_1[n] = e^{j\pi n}h_{\mathrm{LP}}[n]$. Determine and plot the corresponding frequency response $H_1(e^{j\Omega})$. What kind of filters is it? (4%)

(b) Let another filter be with impulse response defined by $h_2[n] = 2h_{\mathrm{LP}}[n]\cos(\pi n/2)$. Determine and plot the corresponding frequency response $H_2(e^{j\Omega})$. What kind of filters is it? (4%)

(c) Is the ideal LPF physically realizable? Why? (3%)

6. Suppose we have two three-point sequences $x[n]$ and $h[n]$ as follows:

$$x[n] = \begin{cases} 1, & 0 \le n \le 2 \\ 0, & \text{otherwise} \end{cases}, \quad h[n] = \begin{cases} -1, & 0 \le n \le 2 \\ 0, & \text{otherwise} \end{cases}.$$

Two periodic sequences $\tilde{x}[n]$ and $\tilde{h}[n]$ are constructed from $x[n]$ and $h[n]$ in the following way:

$$\tilde{x}[n] = \sum_{r=-\infty}^{\infty} x[n+Nr], \quad \tilde{h}[n] = \sum_{r=-\infty}^{\infty} h[n+Nr]$$

(a) Compute $y[n] = x[n] * h[n]$ (linear convolution). (3%)

(b) Compute $\tilde{y}[n] = \tilde{x}[n] \circledast \tilde{h}[n]$ (periodic convolution) for N=3. (3%)

(c) How should we choose N such that $\tilde{y}[n]$ is equal to $y[n]$ for $0 \le n \le N-1$? Explain your answer briefly. (4%)

(d) Describe how to compute $\tilde{y}[n]$ in (c) by using the discrete Fourier transform (DFT) and the inverse DFT. (3%)

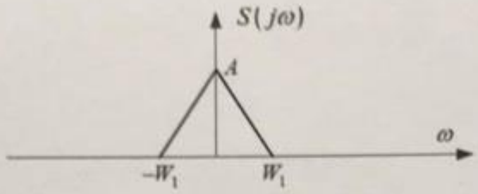
7. Compute the Fourier transform or the inverse Fourier transform of each of the following signals:

$\left(\frac{1}{2}\right)^{n-1}$ $\left(\frac{1}{2}\right)^{-1}$ 2

(a) $x[n]=(1/2)^{-n}u[-n-1]$. (6%)

(b) $X(e^{j\Omega})=\dfrac{1}{\left(1-(1/3)\cdot e^{-j\Omega}\right)^2}$. (6%)

8. Consider the uniform impulse train $p(t)=\sum_{k=-\infty}^{\infty}\delta(t-kT)$ and a continuous-time signal $s(t)$ with Fourier transform $S(j\omega)$ as shown below.



(a) Plot the Fourier transform (or spectrum) of $r(t)=s(t)p(t)$ for $T=5\pi/(4W_1)$. Is it possible to recover $s(t)$ from $r(t)$? Why? (4%)

(b) Plot the Fourier transform (or spectrum) of $r(t)=s(t)p(t)$ for $T=3\pi/(4W_1)$. Is it possible to recover $s(t)$ from $r(t)$? Why? (4%)

(c) Determine the maximum T ($T_{\max}$) such that $s(t)$ can be recovered from $r(t)$. (3%)

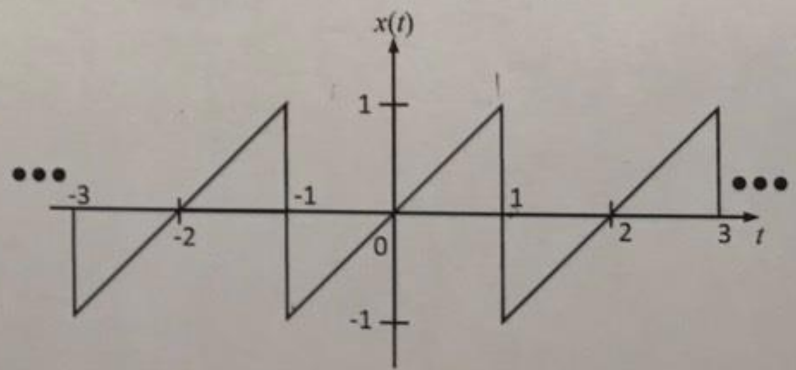
9. Consider a causal LTI system described by the difference equation $y[n]+(1/2)\cdot y[n-1]=x[n]$.

(a) Determine the frequency response $H(e^{j\Omega})$ of this system. (5%)

(b) Determine the output $y[n]$ of the system to the input $x[n]=(1/2)^n u[n]$. (6%)

10. Consider the following continuous-time periodic signal $x(t)$:

$\frac{2\pi}{2}=1$



(a) Why $x(t)$ has a Fourier series representation? (3%)

(b) Denote the Fourier series representation by $x_F(t)=\sum_{k=-\infty}^{\infty}a_k e^{jk\omega_0 t}$. Determine the corresponding Fourier series coefficients a_k. (6%)

(c) Is $x_F(t)$ equal to $\tilde{x}(t)$ for all t? Explain your answer. (3%)