1. (a)

$$x[n]=\sin(\frac{\pi n}{4})+\cos(\frac{\pi n}{2})=\frac{1}{2j}(e^{j\frac{\pi n}{4}}-e^{-j\frac{\pi n}{4}})+\frac{1}{2}(e^{j\frac{\pi n}{2}}+e^{-j\frac{\pi n}{2}})$$

$$y[n]=\frac{1}{2j}[H(e^{j\frac{\pi}{4}})e^{j\frac{\pi n}{4}}-H(e^{-j\frac{\pi}{4}})e^{-j\frac{\pi n}{4}}]+\frac{1}{2}[H(e^{j\frac{\pi}{2}})e^{j\frac{\pi n}{2}}-H(e^{-j\frac{\pi}{2}})e^{-j\frac{\pi n}{2}}]$$

$$=\frac{1}{2j}[2(1+j)e^{j\frac{\pi n}{4}}-2(1-j)e^{-j\frac{\pi n}{4}}]+\frac{1}{2}[\frac{4}{3}e^{j\frac{\pi n}{2}}-\frac{4}{3}e^{-j\frac{\pi n}{2}}]$$

$$=2\sin(\frac{\pi n}{4})+2\cos(\frac{\pi n}{4})+\frac{4}{3}\cos(\frac{\pi n}{2})$$

(b)

$$a_k=\frac{1}{N}\sum_{k=<N>}x[\mathrm{n}]e^{-jk\Omega_0 n}=\frac{1}{N}=\frac{1}{4}$$

$$y[n]=\sum_{k=<4>}\frac{1}{4}H(\mathrm{e}^{jk\frac{\pi}{2}})\mathrm{e}^{jk\frac{\pi}{2}n}$$

$$b_0=0, b_1=\frac{1}{3}, b_2=\frac{1}{3}, b_3=0$$

$$y[n]=\frac{1}{3}e^{j\frac{\pi}{2}n}+\frac{1}{3}e^{-j\frac{\pi}{2}n}=\frac{2}{3}e^{j\pi n}\cos\frac{\pi}{2}n$$

2.

(a)

$$H_A(jw)=1-\frac{2}{5+jw}, h_A(\mathrm{t})=\delta(t)-2e^{-5t}u(\mathrm{t})$$

$$\text{(b)}\ \mathrm{H}_B(jw)=\left[\frac{2+jw}{(3+\mathrm{jw})(1+\mathrm{jw})}\right]$$

$$\text{(c)}\ \frac{d^2y(\mathrm{t})}{dt^2}+6\frac{y(\mathrm{t})}{dt}+5y(\mathrm{t})=\frac{x(\mathrm{t})}{dt}+2x(\mathrm{t})$$

3.

(a)

$$\alpha_3 = \frac{1}{2}e^{j\frac{2}{3}\pi}, \alpha_{-3} = \frac{1}{2}e^{-j\frac{2}{3}\pi}$$

(b)

$$\mathrm{b}_0 = 1, b_1 = \frac{1}{2j}, b_{-1} = \frac{-1}{2j}$$

(c)

$$c_k = \sum_{l=<8>} a_l b_{k-l}$$

$$c_2 = \frac{-e^{j\frac{2\pi}{3}}}{4j}, c_{-2} = \frac{e^{-j\frac{2\pi}{3}}}{4j}, c_3 = \frac{e^{j\frac{2\pi}{3}}}{2}, \mathrm{c}_{-3} = \frac{e^{-j\frac{2\pi}{3}}}{2}, c_4 = \frac{1}{4j}(\mathrm{e}^{j\frac{2\pi}{3}} - \mathrm{e}^{-j\frac{2\pi}{3}})$$

4. (1) .

$\because x(t)$ is real and even

$$\therefore \mathrm{Im}(X(j\omega)) = 0 \quad \Rightarrow \tan^{-1}\left\{\frac{\mathrm{Im}(X(j\omega))}{\mathrm{Re}(X(j\omega))}\right\} = 0$$

(2) $\int_{-\infty}^{\infty} |X(j\omega)|^2 \, d\omega = 2\pi \int_{-\infty}^{\infty} |x(t)|^2 \, dt = 4\pi \int_{1}^{4} |t-1|^2 \, dt = 4\pi \cdot 9 = 36\pi$

(3) $\int_{-\infty}^{\infty} X(j\omega) e^{j2\omega} d\omega = 2\pi x(2) = 2\pi$

5.

(1) (3%)

$$H_1(e^{j\Omega}) = H_{lp}(e^{j(\Omega-\pi)})$$

$$H_1(e^{j\Omega}) = \begin{cases} 0, & |\Omega| < 0.8\pi, \\ 1, & 0.8\pi \le |\Omega| \le \pi. \end{cases} \Rightarrow \text{HPF}.$$

(2) (4%)

$$H_2(e^{j\Omega}) = H_{lp}(e^{j\Omega}) * [\delta(\Omega - 0.5\pi) + \delta(\Omega + 0.5\pi)]$$

$$H_2(e^{j\Omega}) = \begin{cases} 0, & |\Omega| < 0.3\pi, \\ 1, & 0.3\pi \le |\Omega| \le 0.7\pi, \Rightarrow \text{BPF}. \\ 0, & 0.7\pi < |\Omega| \le \pi. \end{cases}$$

(3) NO! The reasons are infinite length and non-causal property of $h_{lp}[n]$. (3%)

6. .

(a) $y[n]=\begin{cases} -1, n=0,4 \\ -2, n=1,3 \\ -3, n=2 \\ 0, \text{otherwise} \end{cases}$

(b) $y[n]=\begin{cases} -3, 0 \le n \le 2 \\ 0, \text{otherwise} \end{cases}$

(c) $N=3+3-1=5$

(d)Calculate the 5-point DFTs X[k] and H[k] of x[n] and h[n].
Multiply these 5-point DFTs together to obtain Y[k]=X[k]H[k].
Calculate the inverse DFT of Y[k] to get y[n].

7. .

(a) $X(e^{j\Omega})=\dfrac{\frac{1}{2}e^{j\Omega}}{1-\frac{1}{2}e^{j\Omega}}$

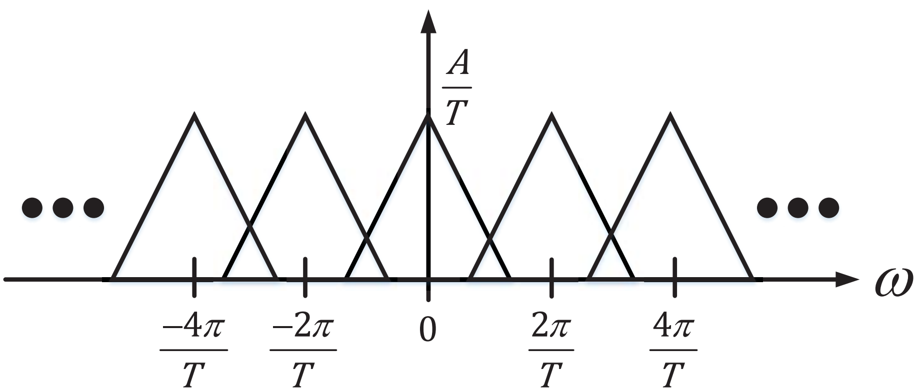
(b) $x[n]=(n+1)(\frac{1}{3})^n u[n]$

8. .

(a)

$$r(t)=s(t)p(t)\xrightarrow{F\{\bullet\}}R(j\omega)=\frac{1}{2\pi}S(j\omega)*P(j\omega)$$

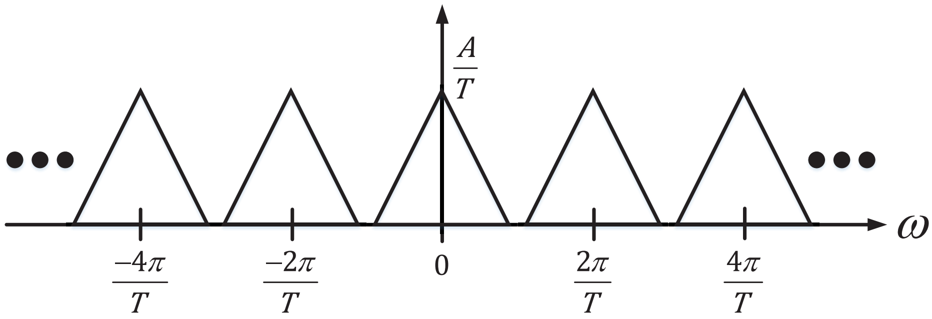
$$\therefore R(j\omega)=\frac{1}{T}S(j\omega)*\sum_{k=-\infty}^{\infty}\delta\left(\omega-\frac{2\pi k}{T}\right)=\frac{1}{T}\sum_{k=-\infty}^{\infty}S\left(j\left(\omega-\frac{2\pi k}{T}\right)\right)$$



$\because \frac{2\pi}{T} < 2W_1$, there exists aliasing, thus recovery is impossible

(b).

$$\frac{2\pi}{T} = 2\pi \times \frac{4W_1}{3\pi} = \frac{8}{3}W_1 > 2W_1$$



Since $\frac{2\pi}{T} > 2W_1$, so perfect recovery is possible, since it satisfies sampling theorem

(c)

$$T_{\max} = \frac{\pi}{w_1}$$

9.

(a)

$$\frac{1}{1+\frac{1}{2}e^{-j\Omega}}$$

(b)

$$\frac{1}{2}(\frac{1}{2})^n u[n] + \frac{1}{2}(-\frac{1}{2})^n u[n]$$

10.(

(a) $\int_T |x(t)|^2\, dt < \infty$ it's period.

(b) $a_k = \frac{1}{T}\int_T te^{-jk\pi t} dt = \frac{1}{-jk\pi}\cos(k\pi) - \frac{j}{k^2\pi^2}\sin(\mathrm{k}\,\pi)$

(c) NO