

## Solution of Quiz #2

Nov. 13, 2014

1. Consider a continuous-time LTI system with impulse response  $h(t)$  whose Fourier transform  $H(j\omega)$  is shown in Fig. 1. Determine the output  $y(t)$  of the system when the input is

$$x(t) = 1 + 2 \cos(2\pi t) + 4 \cos^2(4\pi t) + 8 \cos(2\pi t) \cos(5\pi t).$$

(Note:  $H(j\omega) = \int_{-\infty}^{\infty} h(t)e^{-j\omega t} dt$ )

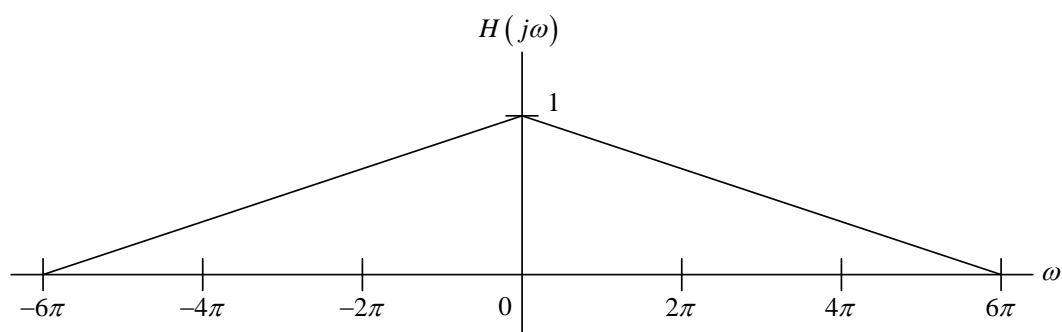


Fig. 1

Answer:

$$x(t) = 1 + 2 \cos(2\pi t) + 2 + 2 \sin(8\pi t) + 4 \cos(7\pi t) + 4 \cos(3\pi t)$$

$$= 3 + e^{j2\pi t} + e^{-j2\pi t} + \frac{1}{j} e^{j8\pi t} - \frac{1}{j} e^{-j8\pi t} + 2e^{j7\pi t} + 2e^{-j7\pi t} + 2e^{j3\pi t} + 2e^{-j3\pi t}$$

$$y(t) = 3H(e^{j\omega})\Big|_{\omega=0} + e^{j2\pi t} H(e^{j\omega})\Big|_{\omega=2\pi} + e^{-j2\pi t} H(e^{j\omega})\Big|_{\omega=-2\pi} + \frac{1}{j} e^{j8\pi t} H(e^{j\omega})\Big|_{\omega=8\pi}$$

$$- \frac{1}{j} e^{-j8\pi t} H(e^{j\omega})\Big|_{\omega=-8\pi} + 2e^{j7\pi t} H(e^{j\omega})\Big|_{\omega=7\pi} + 2e^{-j7\pi t} H(e^{j\omega})\Big|_{\omega=-7\pi}$$

$$+ 2e^{j3\pi t} H(e^{j\omega})\Big|_{\omega=3\pi} + 2e^{-j3\pi t} H(e^{j\omega})\Big|_{\omega=-3\pi}$$

$$= 3 + \frac{4}{3} \cos(2\pi t) + 2 \cos(3\pi t)$$