Reference Solutions of Homework # 8

1.

(1) (10%)

$$x_1[n] = 2^n u[-n]$$

$$X_1(z) = \sum_{n=-\infty}^{\infty} x_1[n]z^{-n} = \sum_{n=-\infty}^{0} 2^n z^{-n} = \sum_{n=0}^{\infty} 2^{-n} z^n = \frac{1}{1-\frac{1}{2}z} = \frac{-2z^{-1}}{1-2z^{-1}}, \quad |z| < 2$$

$$x_2[n] = (\frac{1}{4})^n u[n-1]$$

$$X_2(z) = \sum_{n=-\infty}^{\infty} x_2[n]z^{-n} = \sum_{n=1}^{\infty} (\frac{1}{4})^n z^{-n} = \sum_{n=0}^{\infty} (\frac{1}{4})^{n+1} z^{-n-1} = \frac{\frac{1}{4}z^{-1}}{1-\frac{1}{4}z^{-1}}, \quad |z| > \frac{1}{4}$$

$$X(z) = X_1(z) + X_2(z) = -\frac{2z^{-1}}{1-2z^{-1}} + \frac{\frac{1}{4}z^{-1}}{1-\frac{1}{4}z^{-1}}, \quad \frac{1}{4} < |z| < 2$$

(2) (10%)

$$x_1[n] = (\frac{1}{2})^{|n|} = (\frac{1}{2})^n u[n] + 2^n u[-n-1]$$

$$(\frac{1}{2})^n u[n] \xrightarrow{z} \frac{1}{1-\frac{1}{2}z^{-1}}, \quad |z| > \frac{1}{2}$$

$$2^n u[-n-1] \xrightarrow{z} -\frac{1}{1-2z^{-1}}, \quad |z| < 2$$

$$X_1(z) = \frac{1}{1-\frac{1}{2}z^{-1}} - \frac{1}{1-2z^{-1}}, \quad \frac{1}{2} < |z| < 2$$

$$x[n] = nx_1[n]$$

$$X(z) = -z\frac{d}{dz}X_1(z) = \frac{\frac{1}{2}z^{-1}}{(1-\frac{1}{2}z^{-1})^2} + \frac{2z^{-1}}{(1-2z^{-1})^2}, \quad \frac{1}{2} < |z| < 2$$

2.

(1) (10%)

This pole-zero plot exists “two” two-sided impulse responses.

The two two-sided impulse responses corresponding its ROC as follows:

$$ROC_1 = 0.5 < |z| < 2$$

$$ROC_1 = 2 < |z| < 3$$

(2) (10%)

$$H_2(z) = \frac{\alpha \prod_i (1 - a_i z^{-1})}{\beta \prod_i (1 - b_i z^{-1})} (1 - 2z^{-1})(1 + 3z^{-1})$$

Where α , β and a_i are arbitrary complex number, and $|b_i| < 1$.

3.

(1) (10%)

$$y[n] = x[n] + \frac{9}{8} x[n-1] - \frac{1}{3} y[n-1] + \frac{2}{9} y[n-2]$$

(2) (10%)

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 + \frac{9}{8} z^{-1}}{1 + \frac{1}{3} z^{-1} - \frac{2}{9} z^{-2}} = \frac{1 + \frac{9}{8} z^{-1}}{(1 + \frac{2}{3} z^{-1})(1 - \frac{1}{3} z^{-1})}$$

$H(z)$ has pole at $z = \frac{1}{3}$ and $z = -\frac{2}{3}$. Since the system is causal, the

ROC has to be $|z| > \frac{2}{3}$. The ROC includes the unit circle and hence the

4.

(1) (5%)

$$h[n] = -\frac{1}{2}(\frac{1}{2})^n u[n] + \frac{3}{2}(-\frac{1}{2})^n u[n].$$

(2) (10%)

$$y[n] = \frac{4}{5}(1 - j)e^{-j\frac{\pi}{2}n}.$$

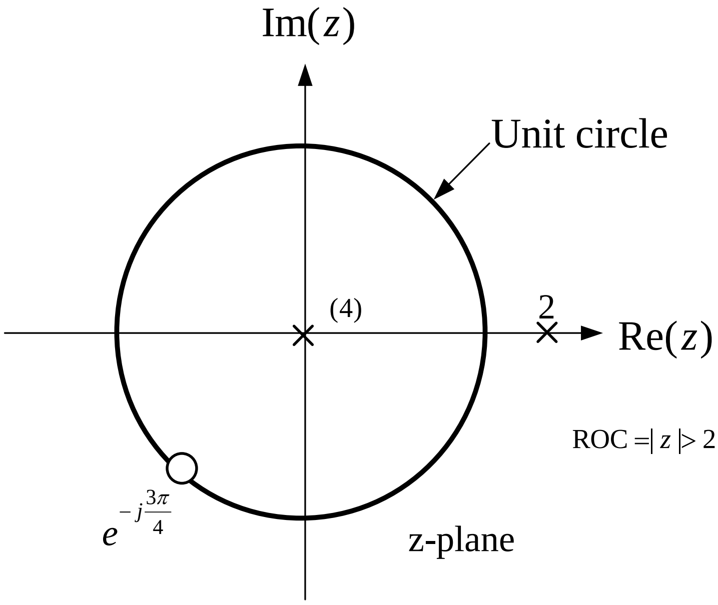
(3) (5%)

No! There exists a pole at the unit circle.

5.

(1) (10%)

$$x_1[n] = x[-n+4] = x[-(n-4)] \Leftrightarrow X_1(z) = z^{-4}X(z^{-1})$$



(2) (10%)

$$x_2[n] = x[n]\cdot(2e^{-j\frac{\pi}{4}})^n \Leftrightarrow X_2(z) = X\left(\frac{1}{2}e^{j\frac{\pi}{4}}z\right)$$