Homework #7

(Due by 14:20, December 31, 2014)

1. Determine the bilateral Laplace transform and the corresponding region of convergence (ROC) or the inverse Laplace transform for the following signals:

   (1) $x(t) = e^{-t}(\sin t)u(t)$. (10%)

   (2) $X(s) = \dfrac{d}{ds}\left(\dfrac{e^{-3s}}{s}\right)$ with ROC $\mathrm{Re}\{s\} > 0$. (10%)

2. Consider a continuous-time linear time-invariant (LTI) system with system function

$$H(s) = \frac{s^2 - 2s + 1}{s^2 - s - 2}.$$

   (1) Plot the poles and zeros of $H(s)$, and indicate all possible ROCs. (6%)

   (2) For each ROC identified in part (1), specify whether the associated system is stable and/or causal. (4%)

   (3) Determine the impulse response $h_{inv}(t)$ of the corresponding stable inverse system. (10%)

3. Consider a system S characterized by the differential equation

$$\frac{d^3y(t)}{dt^3} + 6\frac{d^2y(t)}{dt^2} + 11\frac{dy(t)}{dt} + 6y(t) = x(t).$$

   (1) Determine the zero-state response of this system for the input $x(t) = e^{-4t}u(t)$. (7%)

   (2) Determine the zero-input response of this system for $t > 0^-$, given that

$$y(0^-) = 1, \quad \left.\frac{dy(t)}{dt}\right|_{t=0^-} = -1, \quad \left.\frac{d^2y(t)}{dt^2}\right|_{t=0^-} = 1. \text{ (6\%)}$$

   (3) Determine the output of S when the input is $x(t) = e^{-4t}u(t)$ and the initial conditions are the same as those specified in (2). (7%)

4. A causal LTI system with impulse response $h(t)$ has the following properties:

   (1) When the input to the system is $x(t) = e^{2t}$ for all t, the output is $y(t) = (1/6)e^{2t}$ for all t.

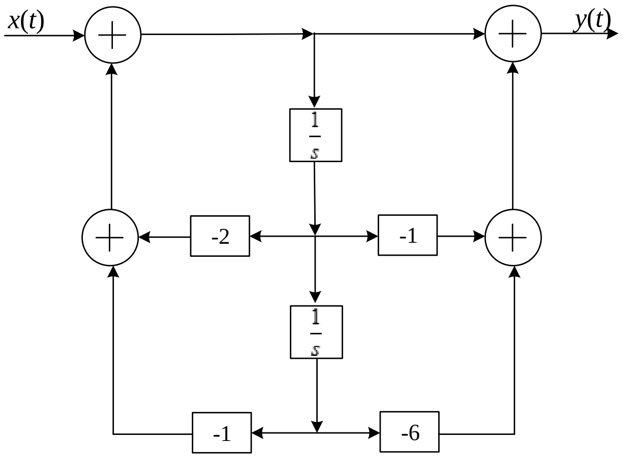
   (2) The impulse response $h(t)$ satisfies the differential equation

$$\frac{dh(t)}{dt} + 2h(t) = (e^{-4t})u(t) + bu(t),$$

   where b is an unknown constant.

   Determine the constant b and the corresponding system function $H(s)$. (10%)

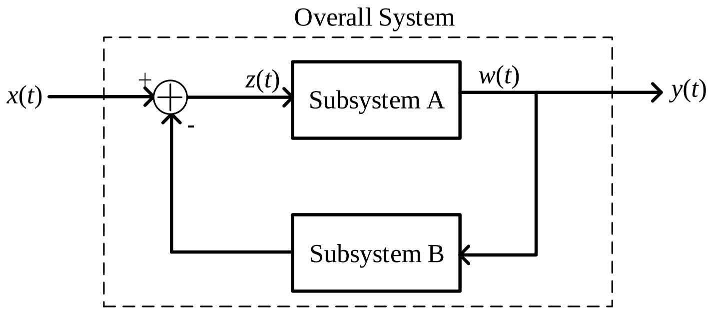
5. The input $x(t)$ and output $y(t)$ of a causal LTI system are related through the block- diagram representation shown in the following figure:



(1) Determine a differential equation relating $y(t)$ and $x(t)$. (5%)

(2) Is this system stable? (5%)

6. Consider the following system:



The input-output relation of the causal Subsystem A is given by

$$\frac{d^2w(t)}{dt^2}+\frac{dw(t)}{dt}+aw(t)=\frac{d^2z(t)}{dt^2}-z(t),$$

and the causal Subsystem B has an impulse response $h_B = e^{-t}u(t)$.

(1) Show that the overall system function can be written as

$$H(s)=\frac{Y(s)}{X(s)}=\frac{H_A(s)}{1+H_A(s)H_B(s)}. \quad (5\%)$$

(2) Determine a such that the overall impulse response is $h(t)=\delta(t)-2e^{-t}u(t)$. (5%)

(3) Find the causal input $x(t)$ that could produce the output $y(t)=e^{-2t}u(t)$. (10%)