Homework No. 6 Solution

1. (15%)

(1) (10%)

From the given information, it is clear that when the input to the system is a complex exponential of frequency Ω_0, the output is a complex exponential of the same frequency but scaled by the $|\Omega_0|$. Therefore, the frequency response of the system is

$$H\left(e^{j\Omega}\right)=|\Omega|, \quad \text{for } 0\le|\Omega|\le\pi.$$

(2) (5%)

Taking the inverse Fourier transform of the frequency response, we obtain

$$\begin{aligned} h[n] &= \frac{1}{2\pi}\int_{-\pi}^{\pi} H\left(e^{j\Omega}\right)e^{j\Omega n}d\Omega \\ &= \frac{1}{2\pi}\int_{-\pi}^{0} -\Omega e^{j\Omega n}d\Omega+\frac{1}{2\pi}\int_{0}^{\pi}\Omega e^{j\Omega n}d\Omega \\ &= \frac{1}{\pi}\int_{0}^{\pi}\Omega\cos\left(\Omega n\right)d\Omega \\ &= \frac{1}{\pi}\left[\frac{\cos\left(n\pi\right)-1}{n^2}\right] \end{aligned}$$

2. (20%)

(1) (10%)

Since the two systems are cascaded, the frequency response of the overall system is

$$\begin{aligned} H\left(e^{j\Omega}\right) &= H_1\left(e^{j\Omega}\right)H_2\left(e^{j\Omega}\right) \\ &= \frac{2-e^{-j\Omega}}{1+\frac{1}{8}e^{-j3\Omega}} \end{aligned}$$

Therefore, the Fourier transforms of the input and output of the overall ststem are related by

$$\frac{Y\left(e^{j\Omega}\right)}{X\left(e^{j\Omega}\right)}=\frac{2-e^{-j\Omega}}{1+\frac{1}{8}e^{-j3\Omega}}$$

Cross-multiplying and taking the inverse Fourier transform, we get

$$y[n]+\frac{1}{8}y[n-3]=2x[n]-x[n-1].$$

(2) (10%)

We may rewrite the overall frequency response as

$$H\left(e^{j\Omega}\right)=\frac{4/3}{1+\frac{1}{2}e^{-j\Omega}}+\frac{\left(1+j\sqrt{3}\right)/3}{1-\frac{1}{2}e^{j120}e^{-j\Omega}}+\frac{\left(1-j\sqrt{3}\right)/3}{1-\frac{1}{2}e^{-j120}e^{-j\Omega}}.$$

Taking the inverse Fourier transform we get

$$h[n]=\frac{4}{3}\left(-\frac{1}{2}\right)^{n}u[n]+\frac{1+j\sqrt{3}}{3}\left(\frac{1}{2}e^{j120}\right)^{n}u[n]+\frac{1-j\sqrt{3}}{3}\left(\frac{1}{2}e^{-j120}\right)^{n}u[n].$$

3. (15%)

(1) (5%) Since $x[n]$ is real and even, $Y\left(e^{j\Omega}\right)=\operatorname{Im}\left\{X\left(e^{j\Omega}\right)\right\}=0.$

(2) (5%) Using time shifting property.

$$y[n]=x[n-4]=|n-4|\left(\frac{1}{3}\right)^{|n-4|}.$$

(3) (5%) Using frequency shifting property.

$$y[n]=x[n]\left(e^{j\frac{\pi}{4}n}+e^{-j\frac{\pi}{4}n}\right)=2|n|\left(\frac{1}{3}\right)^{|n|}\cos\left(\frac{\pi}{4}n\right).$$

4. (15%)

(1) (10%)

$$H(e^{j\Omega})=-e^{-j\Omega}+2e^{-j2\Omega}-2e^{-4j\Omega}+e^{-5j\Omega}=e^{-3j\Omega}(2e^{j\Omega}-2e^{j\Omega}-e^{2j\Omega}+e^{-j2\Omega})$$

$$=e^{-3j\Omega}j(4\sin(\Omega)-2\sin(2\Omega))=e^{-j(3\Omega-\frac{\pi}{2})}(4\sin(\Omega)-2\sin(2\Omega)).$$

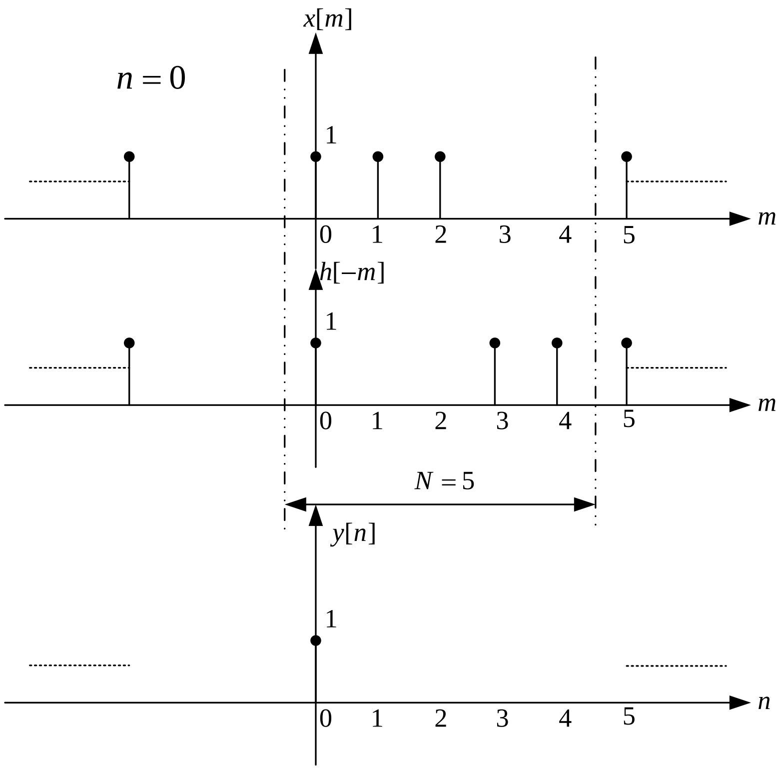
(2) (5%)

Yes! $\because \dfrac{d(-3\Omega-\frac{\pi}{2})}{d\Omega}=-3=\text{constant}.$

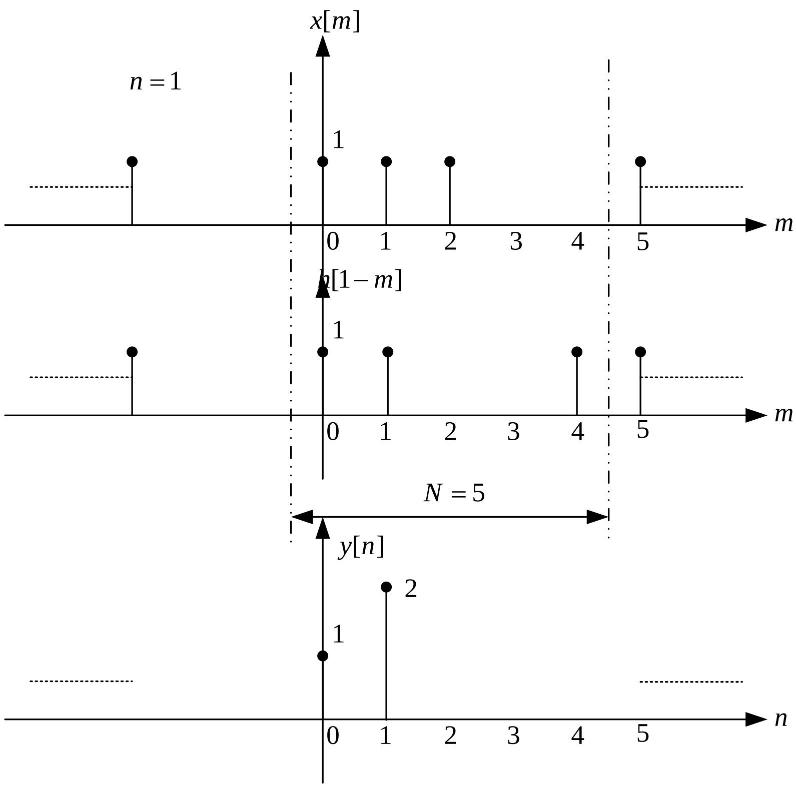
5. (15%)

(1) (5%) $N\geq 3+3-1=5$.

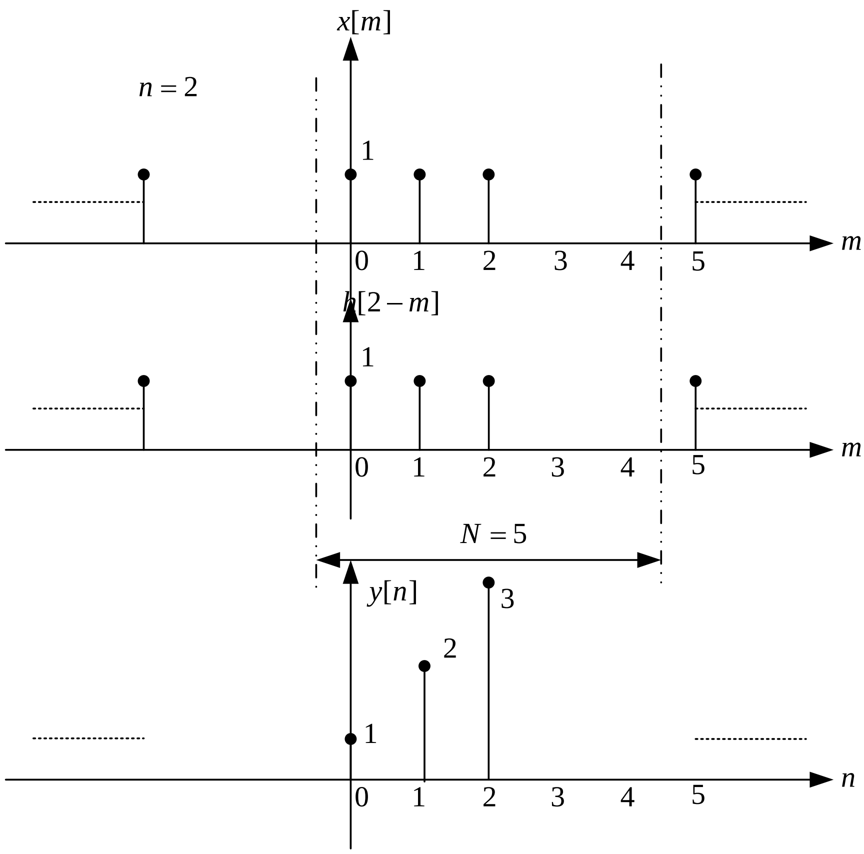
(2) (10%)



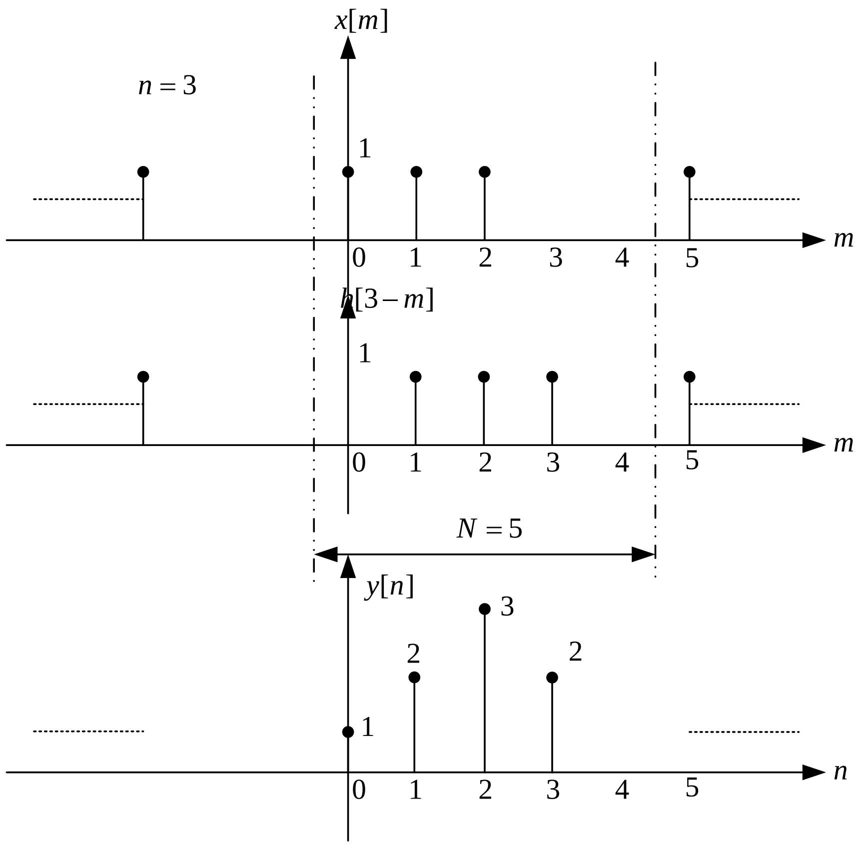
$n=0 \Rightarrow y[0]=1$



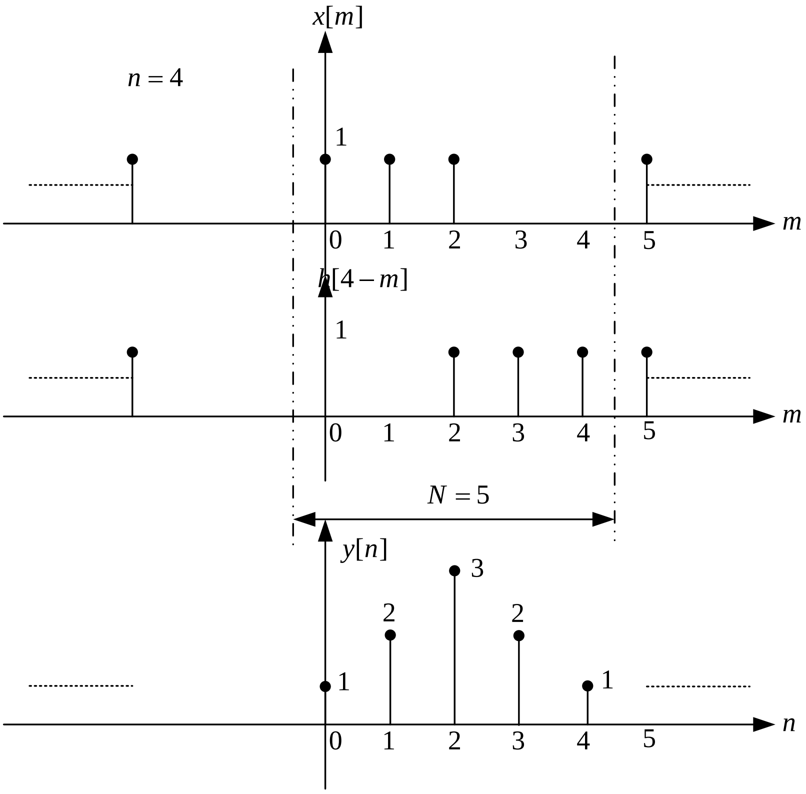
$n=1 \Rightarrow y[1]=2$



$n = 2 \Rightarrow y[2] = 3$



$n = 3 \Rightarrow y[3] = 2$



$n = 4 \Rightarrow y[4] = 1$

6. (20%)

(1) (10%)

The F.S. coefficients of $x[n]$ are $a_k = \frac{1}{4}\sum_{n=0}^{3} x[n] e^{-j2\pi kn/4} = \frac{1}{4}$ for all k .

Thus, the DTFS representation of $x[n]$ is $x[n] = \sum_{k=<4>} \frac{1}{4} e^{j2\pi kn/4}$.

(2) (10%)

$$H(e^{j\Omega}) = 1 + e^{-j\Omega} + e^{-2j\Omega} - e^{j\Omega} - e^{2j\Omega}$$

$$Y(e^{j\Omega}) = \frac{1}{4}(1 + e^{-j\Omega} + e^{-2j\Omega} - e^{j\Omega} - e^{2j\Omega})$$

$$\Rightarrow b_k = \frac{1}{4}(1 + e^{jk\pi/2} + e^{-jk\pi/2})$$