Homework #6

(Due by 19:00, December 16, 2014)

1. A linear time-invariant (LTI) system *S* with impulse response *h*[*n*] and frequency response $H(e^{j\Omega})$ is known to have the property that, when $-\pi \le \Omega_0 \le \pi$,

$$\cos\Omega_0 n \to \Omega_0 \cos\Omega_0 n .$$

   (1) Determine $H(e^{j\Omega})$. (10%)

   (2) Determine *h*[*n*]. (5%)

2. Consider a cascade of two LTI systems with frequency response

$$H_1(e^{j\Omega}) = \frac{2 - e^{-j\Omega}}{1 + 0.5e^{-j\Omega}}$$

   and

$$H_2(e^{j\Omega}) = \frac{1}{1 - 0.5e^{-j\Omega} + 0.25e^{-j2\Omega}} .$$

   (1) Find the difference equation describing the cascade system. (10%)

   (2) Determine the impulse response of the cascade system. (10%)

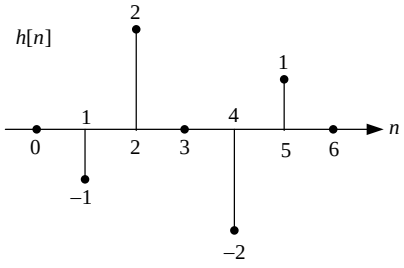
3. Given $x[n] = |n|(1/3)^{|n|} \xleftrightarrow{DTFT} X\left(e^{j\Omega}\right)$. Without evaluating $X(e^{j\Omega})$, find *y*[*n*] for each of the following cases:

   (1) $Y\left(e^{j\Omega}\right) = \mathrm{Im}\left\{X\left(e^{j\Omega}\right)\right\}$. (5%)

   (2) $Y\left(e^{j\Omega}\right) = e^{-j4\Omega} X\left(e^{j\Omega}\right)$. (5%)

   (3) $Y\left(e^{j\Omega}\right) = X\left(e^{j\left(\Omega+\frac{\pi}{4}\right)}\right) + X\left(e^{j\left(\Omega-\frac{\pi}{4}\right)}\right)$. (5%)

4. Consider a discrete-time system with the following impulse response:



   (1) Determine the frequency response $H(e^{j\Omega})$ of the system. (10%)

   (2) Does this system have linear phase? Justify your answer. (5%)

5. Suppose we have two three-point sequences $x[n]$ and $h[n]$ as follows:

$$x[n] = h[n] = \begin{cases} 1, & 0 \le n \le 2 \\ 0, & \text{otherwise} \end{cases}.$$

Two periodic sequences $\tilde{x}[n]$ and $h[n]$ are constructed from $x[n]$ and $h[n]$ in the following way:

$$\tilde{x}[n] = \sum_{r=-\infty}^{\infty} x[n+Nr],\ h[n] = \sum_{r=-\infty}^{\infty} h[n+Nr].$$

(1) Let $\tilde{y}[n] = \tilde{x}[n] \circledast \tilde{h}[n]$ (periodic convolution). How should we choose N such that $y[n] = x[n] * h[n]$ (linear convolution) is equal to $\tilde{y}[n]$ for $0 \le n \le N-1$. (5%)

(2) Compute $y[n]$ by using periodic convolution. (10%)

6. The input signal to a discrete-time LTI system is given by $x[n] = \sum_{k=-\infty}^{\infty} \delta[n-4k]$.

(1) Determine the discrete-time Fourier series of $x[n]$. (10%)

(2) Consider a discrete-time LTI system with impulse response given by

$$h[n] = \begin{cases} 1, & 0 \le n \le 2 \\ -1, & -2 \le n \le -1 \\ 0, & \text{otherwise} \end{cases}.$$

Determine the Fourier series coefficients of the output signal $y[n]$. (10%)