Reference Solutions of Homework # 5

1. (15%)

From Table 3.2, we know that if

$$x[n] \overset{FS}{\longleftrightarrow} a_k ,$$

then

$$(-1)^n x[n] = e^{j(2\pi/N)(N/2)n} x[n] \overset{FS}{\longleftrightarrow} a_{k-N/2} .$$

In this case, $N = 8$. Therefore,

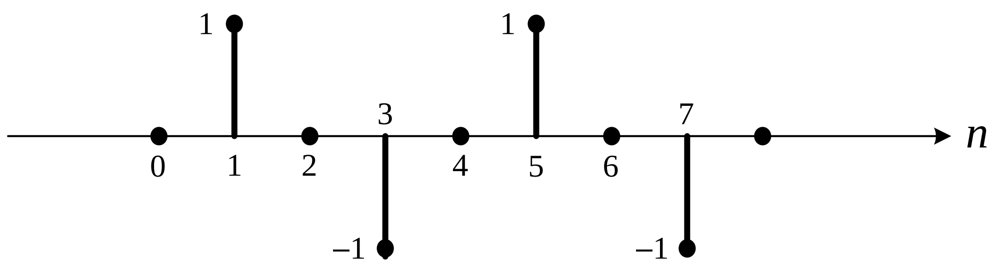
$$(-1)^n x[n] \overset{FS}{\longleftrightarrow} a_{k-4} .$$

Since it is given that $a_k = -a_{k-4}$, we have

$$x[n] = -\left(-1\right)^n x[n] .$$

This implies that $x[0] = x[\pm 2] = x[\pm 4] = \cdots = 0$.

We are also given that $x[1] = x[5] = \cdots = 1$ and $x[3] = x[7] = -1$. Therefore, one period of $x[n]$ is as shown below.



2. (15%)

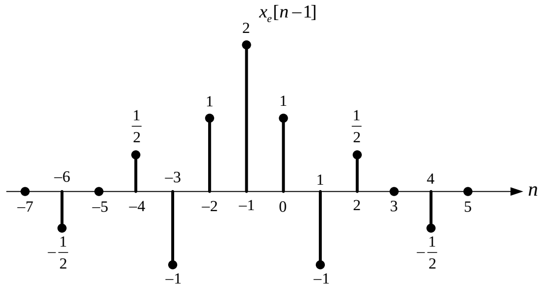
If the inverse Fourier transform of $X(e^{j\Omega})$ is $x[n]$, then

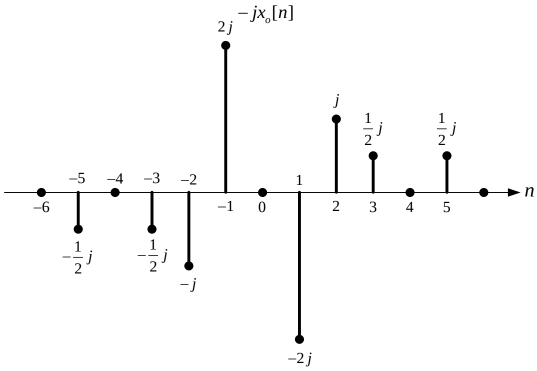
$$x_e[n] = Even\{x[n]\} = \frac{x[n] + x[-n]}{2} \overset{FT}{\longleftrightarrow} A(\Omega)$$

and

$$x_o[n] = Odd\{x[n]\} = \frac{x[n] - x[-n]}{2} \overset{FT}{\longleftrightarrow} jB(\Omega)$$

Therefore, the inverse Fourier transform of $B(\Omega)$ is $-jx_o[n]$. Also, the inverse Fourier transform of $A(\Omega)e^{j\Omega}$ is $x_e[n+1]$. Therefore, the time function corresponding to the inverse Fourier transform of $B(\Omega) + A(\Omega)e^{j\Omega}$ will be $x_e[n+1] - jx_o[n]$. This is as shown below.





3.

(1) (5%)

$$X\left(e^{j0}\right) = \sum_{n=-\infty}^{\infty} x[n] = 6$$

(2) (5%)

Note that $y[n] = x[n+2]$ is an even signal. Therefore, $Y(e^{j\Omega})$ is real and even. This implies that $\measuredangle Y(e^{j\Omega}) = 0$. Furthermore, from the time shifting property of the Fourier transform we have $Y(e^{j\Omega}) = e^{j2\Omega} X(e^{j\Omega})$. Therefore, $\measuredangle X(e^{j\Omega}) = e^{-j2\Omega}$.

(3) (5%)

$$2\pi x[0] = \int_{-\pi}^{\pi} X(e^{j\Omega}) d\Omega$$

$$\int_{-\pi}^{\pi} X(e^{j\Omega}) d\Omega = 4\pi$$

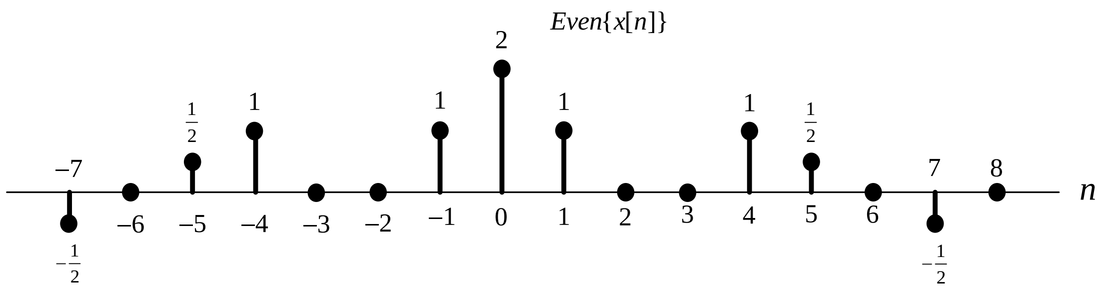
(4) (5%)

$$X(e^{j\pi}) = \sum_{n=-\infty}^{\infty} x[n](-1)^n = 2$$

(5) (5%)

$$Even\{x[n]\} \xleftrightarrow{FT} \mathrm{Re}\{X(e^{j\Omega})\}.$$

Therefore, the desired signal is $Even\{x[n]\} = (x[n] + x[-n])/2$. This is as shown below.



(6) (5%)

(i) From Parseval's theorem we have

$$\int_{-\infty}^{\infty} \left|X(e^{j\Omega})\right|^2 d\Omega = 2\pi \sum_{n=-\infty}^{\infty} \left|x[n]\right|^2 = 28\pi$$

(ii) Using the differentiation in frequency property of the Fourier transform we obtain

$$nx[n] \xleftrightarrow{FT} j\frac{dX(e^{j\Omega})}{d\Omega}.$$

Again using Parseval's theorem, we obtain

$$\int_{-\infty}^{\infty} \left|\frac{X(e^{j\Omega})}{d\Omega}\right|^2 d\Omega = 2\pi \sum_{n=-\infty}^{\infty} \left|n\right|^2 \left|x[n]\right|^2 = 316\pi .$$

4.

(1) (10%)

Since $x[n] \xleftrightarrow{F.S.} a_k$ and $x^*[n] \xleftrightarrow{F.S.} a^*_{-k}$.

By using the convolution property, we have:

$$x[n]x^*[n] = \left|x[n]\right|^2 \xleftrightarrow{F.S.} b_k = \sum_{l=<N>} a_l a^*_{l+k} .$$

(2) (10%)

From (1), it is clear that the answer is yes.

5.

(1) (10%)

$$\begin{aligned} X(e^{j\Omega}) &= \sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n} \\ &= e^{j2\Omega} + 2e^{j\Omega} + 3 + 3e^{-j\Omega} + 2e^{-j2\Omega} + e^{-j3\Omega} \\ &= e^{-j\frac{1}{2}\Omega}(e^{j\frac{5}{2}\Omega} + 2e^{j\frac{3}{2}\Omega} + 3e^{j\frac{1}{2}\Omega} + 3e^{-j\frac{1}{2}\Omega} + 2e^{-j\frac{3}{2}\Omega} + e^{-j\frac{5}{2}\Omega}) \\ &= 2e^{-j\frac{1}{2}\Omega}(\cos(\frac{5}{2}\Omega) + 2\cos(\frac{3}{2}\Omega) + 3\cos(\frac{1}{2}\Omega)) \end{aligned}$$

(2) (10%)

$$Na_k = X(e^{jk\frac{2\pi}{N}}) \Rightarrow a_k = \frac{1}{N}X(e^{jk\frac{2\pi}{N}})$$

$$
\begin{aligned}
a_k &= \frac{1}{6}[2e^{-j\frac{1}{2}\Omega}(\cos(\frac{5}{2}\Omega) + 2\cos(\frac{3}{2}\Omega) + 3\cos(\frac{1}{2}\Omega))] \\
&= \frac{1}{3}e^{-jk\frac{\pi}{6}}(\cos(\frac{5\pi k}{6}) + 2\cos(\frac{\pi k}{2}) + 3\cos(\frac{\pi k}{6}))
\end{aligned}
$$