Homework #5

(Due by 17:30, December 04, 2014)

1. Suppose we are given the following information about a periodic signal $x[n]$ with period 8 and Fourier coefficients a_k:

   (1) $a_k = -a_{k-4}$.

   (2) $x[2n+1] = (-1)^n$.

   Sketch one period of $x[n]$. (15%)

2. Consider the signal depicted in Figure 1. Let the discrete-time Fourier transform of this signal be written in rectangular form as

$$X\left(e^{j\Omega}\right) = A(\Omega) + jB(\Omega).$$

   Sketch the function of time corresponding to the discrete-time Fourier transform

$$Y\left(e^{j\Omega}\right) = \left[B(\Omega) + A(\Omega)e^{j\Omega}\right]. \text{ (15\%)}$$

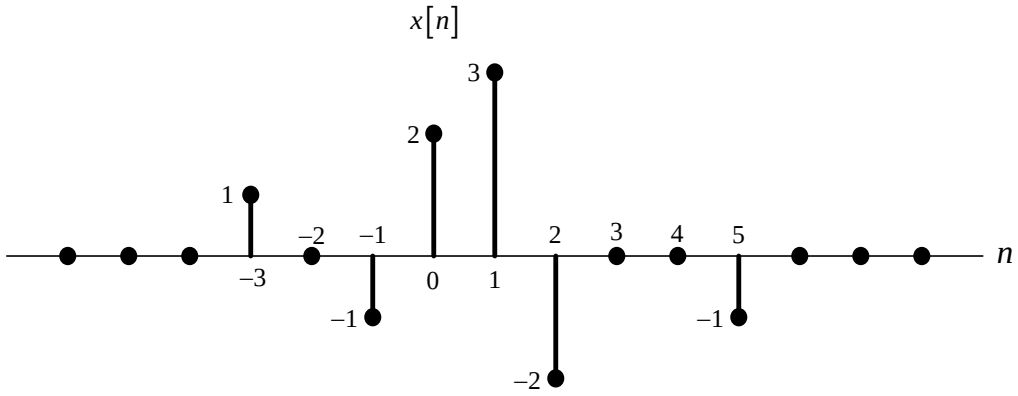


Figure 1

3. Let $X(e^{j\Omega})$ denote the Fourier transform of the signal $x[n]$ depicted in Figure 2. Perform the following calculations without explicitly evaluating $X(e^{j\Omega})$: (30%)

   (1) Evaluate $X(e^{j0})$.

   (2) Find $\measuredangle X(e^{j\Omega})$.

   (3) Evaluate $\int_{-\pi}^{\pi} X(e^{j\Omega}) d\Omega$.

   (4) Find $X(e^{j\pi})$.

   (5) Determine and sketch the signal whose Fourier transform is $\mathrm{Re}\{x(\Omega)\}$.

   (6) Evaluate:

   (i) $\int_{-\pi}^{\pi} \left|X(e^{j\Omega})\right|^2 d\Omega$; (ii) $\int_{-\pi}^{\pi} \left|\frac{dX(e^{j\Omega})}{d\Omega}\right|^2 d\Omega$.

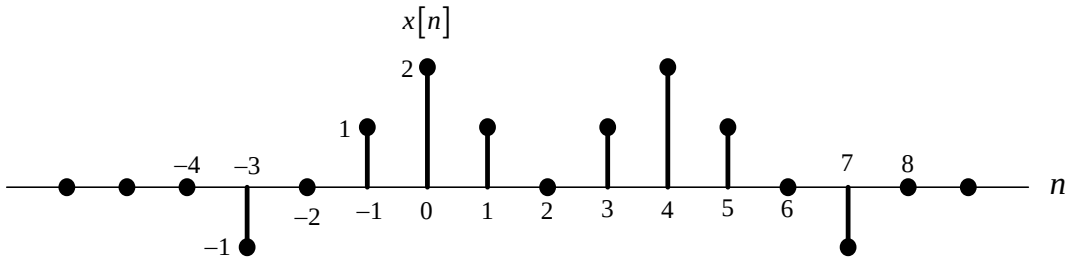


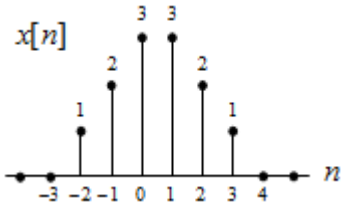
Figure 2

4. Let $x[n]$ be a periodic signal with period N and Fourier coefficients a_k.
   (1) Express the Fourier coefficients b_k of $|x[n]|^2$ in terms of a_k. (10%)
   (2) If the coefficients a_k are real, is it guaranteed that the coefficients b_k are also real? (10%)

5. Let $\tilde{x}[n]$ be a periodic signal with period N. A finite-duration signal $x[n]$ is related to $\tilde{x}[n]$ through

$$x[n]=\begin{cases}\tilde{x}[n], & n_0 \le n \le n_0+N-1\\ 0, & \text{otherwise}\end{cases},$$

for some integer n_0. That is, $x[n]$ is equal to $\tilde{x}[n]$ over one period and zero elsewhere. Consider the following two signals:



$$\tilde{x}[n]=\sum_{k=-\infty}^{\infty} x[n-6k]$$

Let a_k denote the Fourier coefficients of $\tilde{x}[n]$ and let $X\left(e^{j\Omega}\right)$ denote the Fourier transform of $x[n]$.

(1) Determine a closed-form expression for $X\left(e^{j\Omega}\right)$. (10%)
(2) Using the result of part (1), determine an expression for the Fourier coefficients a_k. (10%)