Homework #4

(Due by 17:30, November 20, 2014)

1. Evaluate the quantities for the following signal: (20%)
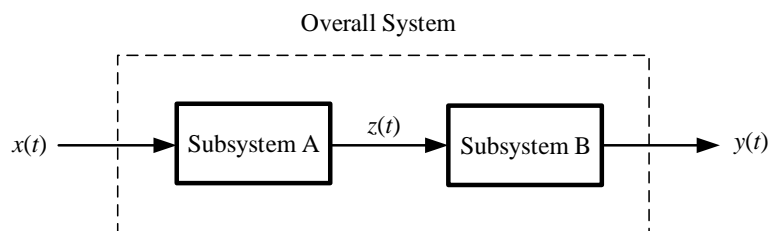
$$X(j\omega) = \mathcal{F}\{x(t)\}$$
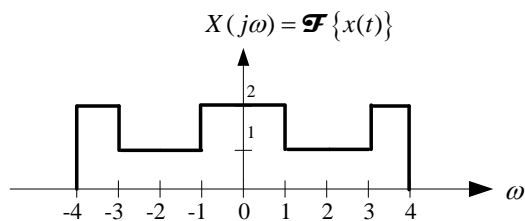
(1) $\int_{-\infty}^{\infty} x(t)dt$.

(2) $\int_{-\infty}^{\infty} |x(t)|^2 \, dt$.

(3) $\int_{-\infty}^{\infty} x(t)e^{j2t}dt$.

(4) $x(0)$.

(5) $\left. \dfrac{dx(t)}{dt} \right|_{t=0}$.

2. Determine the Fourier transform or the inverse Fourier transform of the following signals:

(1) $x(t) = \displaystyle\sum_{k=0}^{2} (-1)^k \sin\left(\frac{2\pi k}{3}t\right)$. (10%)

(2) $X(j\omega) = \dfrac{1}{(a+j\omega)^3}$, $a > 0$. (10%)

3. Consider the following system:

The input-output relation of Subsystem A is given by

$$\frac{d^2 z(t)}{dt^2} - \frac{dz(t)}{dt} - 6z(t) = x(t),$$

and the input-output relation of Subsystem B is given by

$$\frac{dy(t)}{dt} + 6y(t) = \frac{dz(t)}{dt} + bz(t).$$

(1) Determine the frequency response and the impulse response of Subsystem A. (10%)
(2) Determine b such that the overall system is causal. Justify your answer. (10%)

4. Consider an LTI system S with impulse response

$$h(t) = \frac{\sin(4(t-1))}{\pi(t-1)}.$$

Determine the output of S for each of the following inputs: (20%)

(1) $x_1(t) = \cos(6t + \pi/2)$.

(2) $x_2(t) = \sum_{k=0}^{\infty} (\frac{1}{2})^k \sin(3kt)$.

(3) $x_3(t) = \frac{\sin(4(t+1))}{\pi(t+1)}$.

(4) $x_4(t) = (\frac{\sin(2t)}{\pi t})^2$.

5. Consider an LTI system whose response to the input

$$x(t) = [e^{-t} + e^{-3t}]u(t)$$

is

$$y(t) = [2e^{-t} - 2e^{-4t}]u(t).$$

(1) Find the frequency response of this system. (5%)
(2) Determine the impulse response of this system. (10%)
(3) Find the differential equation relating the input and the output of this system. (5%)