Homework #2

(Due by 17:30, October 30, 2014)

1. Consider a discrete-time LTI system whose input $x[n]$ and output $y[n]$ are related by the following difference equation:

$$y[n]-y[n-1]+\frac{1}{4}y[n-2]=\frac{1}{2}x[n],\ y[-1]=1, y[-2]=0. \quad (20\%)$$

   (1) Determine the system output $y[n]$ for $n\geq 0$ and for $n\leq -1$ when the input is $x[n]=u[n]$.

   (2) Determine the particular solution $y^{(p)}[n]$ of the difference equation for $x[n]=(\frac{1}{2})^n u[n]$.

2. (15%)

   (1) Determine a state-variable description for the continuous-time LTI system shown in Fig. 1.

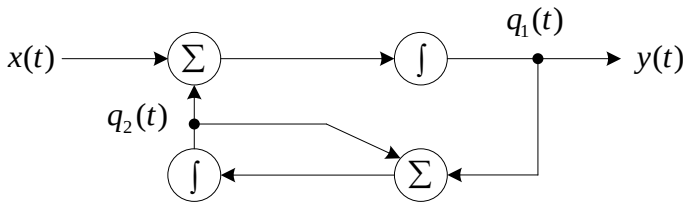

Fig. 1

   (2) Draw the direct form I and direct form II realizations of the system described by

$$y[n]+\frac{1}{4}y[n-1]-2y[n-3]=2x[n-1]-3x[n-2].$$

3. Consider a discrete-time linear time-invariant (LTI) system, where some of its input-output relations are given in Fig. 2. (15%)

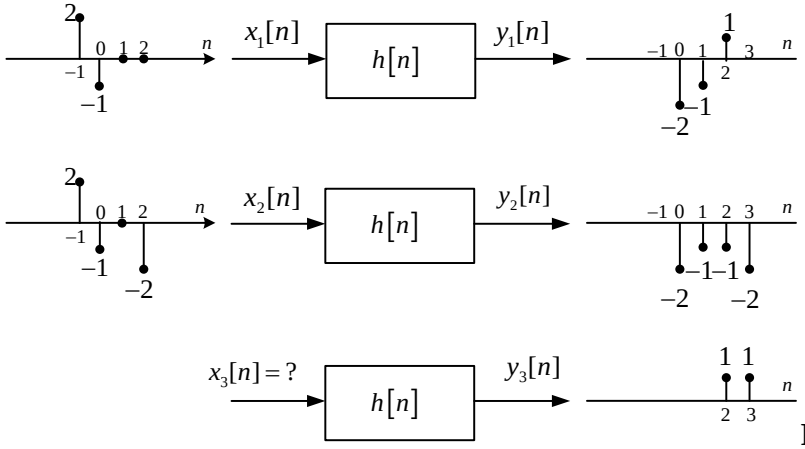

Fig. 2

   (1) What is $x_3[n]$?

   (2) Determine $h[n]$.

4. Consider a continuous-time LTI system with the impulse response $h(t)$ and the input signal $x(t)$ shown in Fig. 3. (15%)

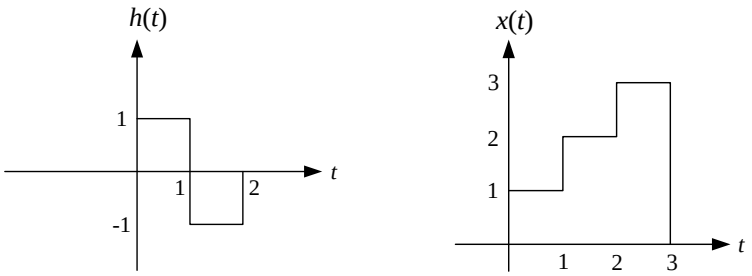

Fig. 3

(1) Determine the step response $s(t)$ of the system.

(2) Express the input signal $x(t)$ in terms of the unit step signal.

(3) Determine the output $y(t)$ for the input signal $x(t)$.

5. (20%)

(1) Consider an LTI system with input and output related through the equation

$$y(t)=\int_{-\infty}^{t} e^{-(t-\tau)}x(\tau-2)\,d\tau.$$

What is the impulse response $h(t)$ for this system.

(2) Determine the response of the system when the input $x(t)$ is as shown in Fig. 4.

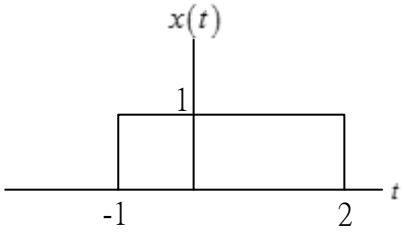

Fig. 4

6. Consider an LTI system described by the following differential equation:

$$\frac{d^2y(t)}{dt^2}-5\frac{dy(t)}{dt}+6y(t)=2x(t)\text{ . (15\%)}$$

Determine the particular solution $y^{(p)}(t)$ for each of the following inputs:

(1) $x(t)=\sin(3t)$.

(2) $x(t)=e^{-3t}+e^{-2t}$.