Homework #1

(Due by 14:20, October 15, 2014)

1. Consider a signal $x(t)$ shown in Fig. 1. Sketch and label carefully each of the following signals: (20%)

   (1) $y(t) = x(2t+5)$;

   (2) $y(t) = [x(t) + x(-t)]u(t)$;

   (3) $y(t) = x(t)[\delta(t+\frac{5}{2}) - \delta(t-\frac{5}{2})]$.

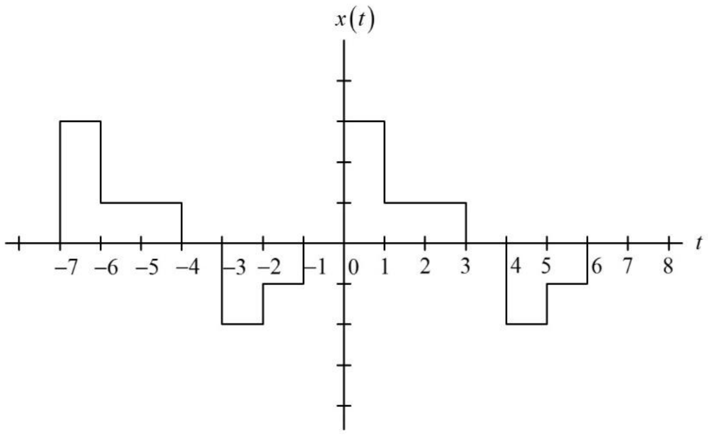

Fig. 1

2. Determine whether or not each of the following signals is periodic; if so, find the fundamental period. (20%)

   (1) $x(t) = \textit{Even part of } \sin(4\pi t)u(t)$;

   (2) $x(t) = \sum_{k=-\infty}^{+\infty} (-0.1)^k \delta(t-2k)$;

   (3) $x[n] = \cos(\frac{\pi}{8}n^2)$;

   (4) $x[n] = \cos(\frac{\pi}{2}n)\cos(\frac{\pi}{4}n)$;

   (5) $x[n] = 2\cos(\frac{\pi}{4}n) + \sin(\frac{\pi}{8}n) - 2\cos(\frac{\pi}{2}n + \frac{\pi}{6})$.

3. Consider a discrete-time signal $x[n]$ whose even and odd parts are denoted by $x_e[n]$ and $x_o[n]$, respectively. Show that

$$\sum_{n=-\infty}^{+\infty} x^2[n] = \sum_{n=-\infty}^{+\infty} x_e^2[n] + \sum_{n=-\infty}^{+\infty} x_o^2[n].\ (10\%)$$

4. Consider a discrete-time signal $x[n]$ and let

$$y_1[n] = x[2n] \text{ and } y_2[n] = \begin{cases} x[n/2], & n \text{ even} \\ 0, & n \text{ odd} \end{cases}.$$

The signals $y_1[n]$ and $y_2[n]$ respectively represent in some sense the speeded up and slowed down versions of $x[n]$. Determine whether each of the following statements is true or false; if true, find the relationship between the fundamental periods of the two signals considered; if false, give a counterexample. (20%)

(1) If $x[n]$ is periodic, then $y_1[n]$ is periodic.

(2) If $y_1[n]$ is periodic, then $x[n]$ is periodic.

(3) If $x[n]$ is periodic, then $y_2[n]$ is periodic.

(4) If $y_2[n]$ is periodic, then $x[n]$ is periodic.

5. The following systems have input $x(t)$ or $x[n]$ and output $y(t)$ or $y[n]$. For each system, determine whether or not it is (i) memoryless, (ii) invertible, (iii) causal, (iv) bounded-input bounded-output stable, (v) time-invariant, and (vi) linear. Justify your answers briefly. Also find the inverse system if the system is invertible. (30%)

(1) $y(t) = \cos(x(t))$;

(2) $y(t) = x(2-t)$;

(3) $y[n] = \sum_{k=-\infty}^{n} x[k+2]$;

(4) $y[n] = \log_{10}(|x[n]|)$.