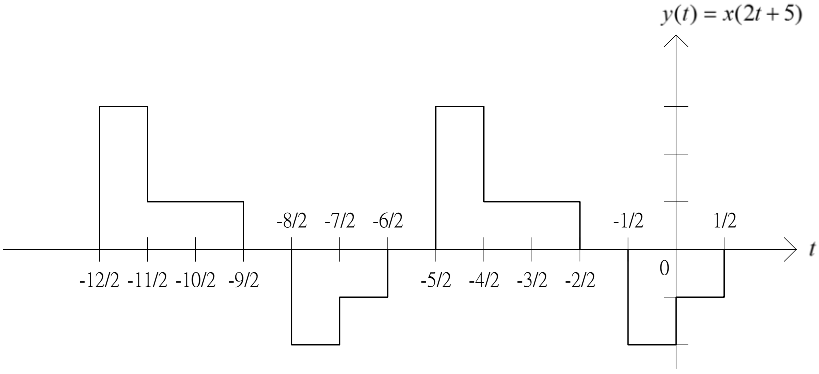
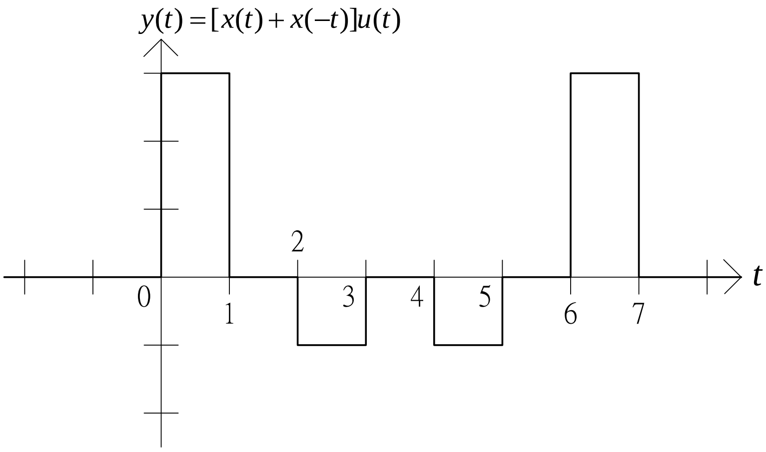
Reference Solutions of Homework # 1

1.

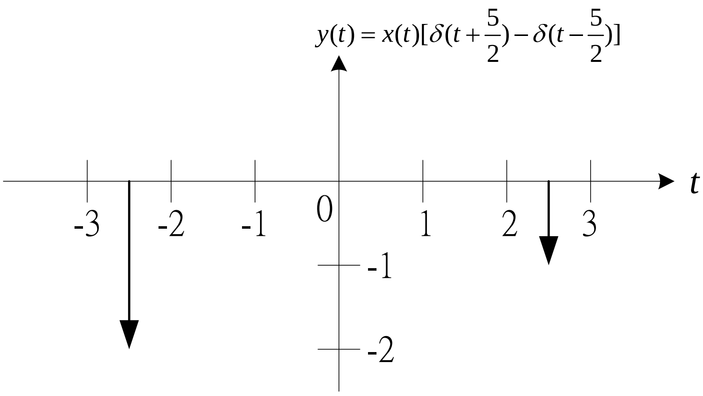
(1)



(2)



(3)



2.

(1) $x(t) = [\sin(4\pi t)u(t) - \sin(4\pi t)u(-t)]/2$. => Not periodic.

(2) Not periodic

(3) Periodic
Fundamental period = 8

(4) $x(t) = (1/2)[\cos(3\pi n/4) + \cos(\pi n/4)]$.

Periodic
Fundamental period = 8

(5) Periodic
Fundamental period = 16

3. Consider

$$\sum_{n=-\infty}^{\infty} x^2[n] = \sum_{n=-\infty}^{\infty} \{x_e[n] + x_o[n]\}^2$$
$$= \sum_{n=-\infty}^{\infty} x_e^2[n] + \sum_{n=-\infty}^{\infty} x_o^2[n] + 2\sum_{n=-\infty}^{\infty} x_e[n]x_o[n].$$

Since $x_e[n]x_o[n]$ is an odd signal, and $\sum_{n=-\infty}^{\infty} x[n] = 0$ if $x[n]$ is odd signal.

Therefore, we can conclude that $2\sum_{n=-\infty}^{\infty} x_e[n]x_o[n] = 0$. Hence, we have

$$\sum_{n=-\infty}^{\infty} x^2[n] = \sum_{n=-\infty}^{\infty} x_e^2[n] + \sum_{n=-\infty}^{\infty} x_o^2[n]$$

4.

(1) True. $x[n] = x[n+N]$, $y_1[n] = y_1[n+N_0]$. i.e. periodic with $N_0 = N/2$ if N is even, and with period $N_0 = N$ if N is odd.

(2) False. $y_1[n]$ periodic does not imply $x[n]$ is periodic. i.e. let

$x[n] = g[n] + h[n]$ where

$$g[n] = \begin{cases} 1, & n \text{ even} \\ 0, & n \text{ odd} \end{cases}, \text{ and } h[n] = \begin{cases} 0, & n \text{ even} \\ (1/2)^n, & n \text{ odd} \end{cases}.$$

Then $y_1[n] = x[2n]$ is periodic but $x[n]$ is clearly not periodic.

(3) True. $x[n+N] = x[n]$; $y_2[n+N_0] = y_2[n]$ where $N_0 = 2N$.

(4) True. $y_2[n+N]=y_2[n]$; $x[n+N_0]=x[n]$ where $N_0=N/2$.

5.

	Memoryless	invertible	Stable	Causal	Linear	Time-invariant
(1)	○	X	○	○	X	○
(2)	X	○	○	X	○	○
(3)	X	○	X	X	○	○
(4)	○	X	X	○	X	○

The inverse system of sub-problem (2): $x(t)=y(-2-t)$

The inverse system of sub-problem (3): $x[n]=y[n-2]-y[n-3]$