7. Consider the finite-length sequence

$$x[n] = 2\delta[n] + \delta[n-1] + \delta[n-3].$$

(1) Compute the five-point DFT $X[k]$. (3%)

(2) If $Y[k] = X^2[k]$, determine the sequence $y[n]$ with five-point inverse DFT for n=0, 1, 2, 3, 4. (4%)

(3) If N-point DFTs are used here, how should we choose N such that $y[n] = x[n] * x[n]$, for $0 \le n \le N-1$. (3%)

8. You are given $x[n] = |n|\left(\frac{1}{3}\right)^{|n|} \xleftrightarrow{DTFT} X(\Omega)$. Without evaluating $X(\Omega)$, find $y[n]$ if:

(1) $Y(\Omega) = \text{Im}\{X(\Omega)\}$. (3%)

(2) $Y(\Omega) = X(\Omega) \circledast X(\Omega + \frac{\pi}{2})$. (3%)

(3) $Y(\Omega) = e^{-j4\Omega}\left[X'(\Omega + \frac{\pi}{4}) + X(\Omega - \frac{\pi}{4})\right]$. (4%)

$x[n] = \sum_{k=0}^{4} X[k] e^{jk\cdot\frac{2\pi}{5}n}$

$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$

9. When the input to an LTI system is

$$x[n] = (\tfrac{1}{2})^n u[n] + \tfrac{1}{4}^n u[n],$$

$X(\omega) = \int_{-\infty}^{0} e^{at} e^{-j\omega t} dt$

$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega$

the output is

$$y[n] = 6(\tfrac{1}{2})^n u[n] + 6(\tfrac{3}{4})^n u[n],$$

(1) Find the frequency response $H(\Omega)$ of the system. (4%)

(2) Find the impulse response $h[n]$ of the system. (4%)

(3) Write the difference equation that characterizes the system. (2%)

10. Consider the signal

$$x[n] = \sin(\frac{\pi n}{8}) - 2\cos(\frac{\pi n}{4}).$$

Suppose that this signal is the input to LTI system with the impulse response

$(2 - \frac{3}{4})(1 - \frac{3}{4})$

$$h[n] = \frac{\sin(\pi n/6)\sin(\pi n/2)}{\pi^2 n^2},$$

(1) Determine and sketch $X(\Omega)$ and $H(\Omega)$. (6%)

(2) Determine the output $y[n]$ of this system. (4%)

(***Hint***: $x[n] = \dfrac{\sin(Wn)}{\pi n}$,0<W<π $\xleftrightarrow{\mathcal{F}} X(\Omega) = \begin{cases} 1, & 0 \le \Omega \le W \\ 0, & W \le \Omega \le \pi \end{cases}$.)