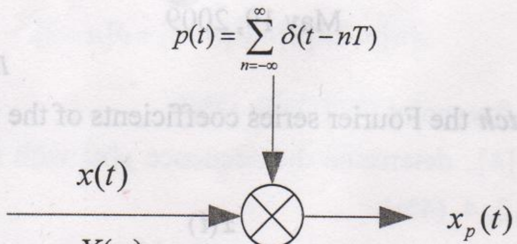
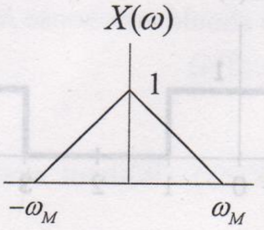
5. Consider the following figure:





(1) Sketch $X_p(\omega)$ for $T=\frac{2\pi}{\omega_M}$. Is it possible to reconstruct $x(t)$ from $x_p(t)$? Why? (5%)

(2) Sketch $X_p(\omega)$ for $T=\frac{\pi}{2\omega_M}$. Is it possible to reconstruct $x(t)$ from $x_p(t)$? Why? (5%)

(Label your axes clearly and carefully!)

6. Let $\tilde{x}[n]$ be a periodic signal with period N. A finite-duration signal $x[n]$ is related to $\tilde{x}[n]$ through

$$x[n]=\begin{cases}\tilde{x}[n], & n_0 \le n \le (n_0+N-1) \\ 0\ , & \text{otherwise}\end{cases}, \quad \text{for some integer } n_0.$$

That is, $x[n]$ is equal to $\tilde{x}[n]$ over one period and zero elsewhere.

(1) If $\tilde{x}[n]$ has Fourier series coefficients a_k and $x[n]$ has Fourier transform $X(\Omega)$, show that

$$a_k=\frac{1}{N}X(e^{j\frac{2\pi k}{N}})$$

regardless of the value of n_0. (6%)

(2) Consider the signals $x[n]=u[n]-u[n-4]$ and $\tilde{x}[n]=\sum_{k=-\infty}^{\infty}x[n-kN]$, where N is a positive integer. Let a_k denote the Fourier coefficients of $\tilde{x}[n]$ and $X(\Omega)$ denote the Fourier transform of $x[n]$. Determine $X(\Omega)$ and a_k. (4%)