Homework No. 5

Due 13:10, Dec. 14, 2011

Please use the A4 format to show your results!

1. Let $x[n]$ be a periodic signal with period N and Fourier coefficients a_k.

(1) Express the Fourier coefficients b_k of $\left|x[n]\right|^2$ in terms of a_k. (10%)

(2) If the coefficients a_k are real, is it guaranteed that the coefficients b_k are also real? (10%)

2. When the impulse train $x[n]=\sum_{k=-\infty}^{\infty}\delta[n-4k]$ is the input to a particular LTI system with frequency response $H\left(e^{j\Omega}\right)$, the output of the system is found to be $y[n]=\cos\left(\frac{5\pi}{2}n+\frac{\pi}{4}\right)$. Determine the values of $H\left(e^{jk\pi/2}\right)$ for k = 0, 1, 2, and 3. (20%)

3. You are given $x[n]=n(1/2)^{|n|}\xleftrightarrow{DTFT}X(\Omega)$. Without evaluating $X(\Omega)$, find $y[n]$ if

(1) $Y(\Omega)=\mathrm{Re}\{X(\Omega)\}$ (5%)

(2) $Y(\Omega)=dX(\Omega)/d\Omega$ (5%)

(3) $Y(\Omega)=X(\Omega)+X(-\Omega)$ (5%)

(4) $Y(\Omega)=e^{-4j\Omega}X(\Omega)$ (5%)

4. Let $x[n]$ and $h[n]$ be the signals with the following Fourier transforms:

$$X\left(e^{j\Omega}\right)=3e^{-j\Omega}+1-e^{j\Omega}+2e^{j3\Omega}$$
$$H\left(e^{j\Omega}\right)=2e^{-j2\Omega}-e^{-j\Omega}+e^{j4\Omega}$$

Determine $y[n]=x[n]*h[n]$. (15%)

5. Consider the finite-length sequence $x[n]=2\delta[n]+\delta[n-1]+\delta[n-3]$.

(1) Compute the five-point DFT $X[k]$. (10%)

(2) If $Y[k]=X^2[k]$, determine the sequence $y[n]$ with five-point inverse DFT for n = 0 ~ 4. (10%)

(3) If N-point DFTs are used here, how should we choose N such that $y[n]=x[n]*x[n]$, for $0\le n\le N-1$. (5%)