

Homework No. 4 Solution

1. (20%)

(1) Fundamental period of $x(t) = T = 4 \Rightarrow \omega_0 = 2\pi/4 = \pi/2$

$$X[0] = \frac{1}{T} \int_T x(t) dt = \frac{1}{4} \int_{-1}^1 x(t) dt = 0.5$$

$$X[k] = \frac{1}{4} \int_{-1}^1 x(t) e^{-jk\omega_0 t} dt = \frac{1}{-j4k\omega_0} e^{-jk\omega_0 t} \Big|_{-1}^1 = \frac{1}{jk2\pi} (e^{jk\omega_0} - e^{-jk\omega_0}) = \frac{\sin\left(\frac{k\pi}{2}\right)}{k\pi}$$

(2) Fundamental period of $y(t) = T = 2 \Rightarrow \omega_0 = 2\pi/2 = \pi$

$$Y[0] = \frac{1}{T} \int_T y(t) dt = 0; \quad y(t) = \sin \pi t = \frac{e^{j\pi t} - e^{-j\pi t}}{2j} \Rightarrow Y[k] = \begin{cases} -\frac{j}{2}, & k=1 \\ \frac{j}{2}, & k=-1 \\ 0, & \text{o.w.} \end{cases}$$

2. (20%)

$$(1) x(t) = \sum_{m=-\infty}^{\infty} X[k] e^{j2\pi kt} = je^{j2\pi t} - je^{-j2\pi t} + e^{j(3)2\pi t} + e^{j(-3)2\pi t} = -2\sin(2\pi t) + 2\cos(6\pi t)$$

$$(2) x(t) = \sum_{k=0}^{\infty} \left(-\frac{1}{3}\right)^k e^{jk\pi t} + \sum_{k=1}^{\infty} \left(-\frac{1}{3}\right)^{-k} e^{-jk\pi t} = \frac{1}{1+\frac{1}{3}e^{j\pi t}} + \frac{\frac{1}{3}e^{-j\pi t}}{1+\frac{1}{3}e^{-j\pi t}}$$

$$= \frac{4}{5+3\cos(\pi t)}$$

3. (30%)

$$(1) X(\omega) = \frac{1}{1+j\omega}, \text{ and } Y(\omega) = \frac{1}{2+j\omega} + \frac{1}{3+j\omega} = \frac{5+2j\omega}{(2+j\omega)(3+j\omega)}$$

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{5+7j\omega-2\omega^2}{(2+j\omega)(3+j\omega)} = 2 - \frac{1}{2+j\omega} - \frac{2}{3+j\omega}$$

$$\therefore h(t) = 2\delta(t) - (e^{-2t} + 2e^{-3t})u(t)$$

$$(2) X(\omega) = \frac{1}{2+j\omega}, \text{ and } Y(\omega) = \frac{2}{(2+j\omega)^2} e^{-j2\omega}$$

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{2}{2+j\omega} e^{-j2\omega} \therefore h(t) = 2e^{-2(t-2)}u(t-2)$$

4. (30%)

(1)

$$\begin{aligned} x(t) &= \sin(2\pi t)e^{-t}u(t) \\ &= \frac{1}{2j}e^{j2\pi t}e^{-t}u(t) - \frac{1}{2j}e^{-j2\pi t}e^{-t}u(t) \end{aligned}$$

$$\begin{aligned} e^{-t}u(t) &\xleftrightarrow{FT} \frac{1}{1+j\omega} \\ e^{j2\pi t}s(t) &\xleftrightarrow{FT} S(j(\omega - 2\pi)) \\ X(j\omega) &= \frac{1}{2j} \left[\frac{1}{1+j(\omega - 2\pi)} - \frac{1}{1+j(\omega + 2\pi)} \right] \end{aligned}$$

(2)

$$\begin{aligned} \frac{\sin(Wt)}{\pi t} &\xleftrightarrow{FT} \begin{cases} 1 & \omega \leq W \\ 0, & \text{otherwise} \end{cases} \\ s_1(t)s_2(t) &\xleftrightarrow{FT} \frac{1}{2\pi}S_1(j\omega) * S_2(j\omega) \\ X(j\omega) &= \begin{cases} 5 - \frac{|\omega|}{\pi} & \pi < |\omega| \leq 5\pi \\ 4 & |\omega| \leq \pi \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

(3)

$$\begin{aligned} \text{Since } \frac{1}{(1+j\omega)^2} &\xleftrightarrow{F.T.} te^{-t}u(t) \quad \text{and} \quad j\omega S(\omega) \xleftrightarrow{F.T.} \frac{d}{dt}s(t) \\ \therefore x(t) &= \frac{d}{dt}[te^{-t}u(t)] = (1-t)e^{-t}u(t) \end{aligned}$$