# Homework No. 3 Solution

1. Homogeneous solution

$$r^2+4=0 \Rightarrow r=\pm j2$$
$$y^h(t)=c1e^{j2t}+c2e^{-j2t}$$

(a) (5%) $x(t)=t$

$$y^p(t)=p_1t+p_2$$

$$4p_1t+4p_2=3 \Rightarrow p_1=0, p_2=\frac{3}{4}$$

$$\therefore y^p(t)=\frac{3}{4}$$

$$\therefore y(t)=y^h(t)+y^p(t)=c_1e^{j2t}+c_2e^{-j2t}+\frac{3}{4}$$

$$\therefore y(t)=b_1\sin(2t)+b_2\cos(2t)+\frac{3}{4}$$

From $y(0^-)=-1, \frac{d}{dt}y(t)|_{t=0^-}=1$

We get $\Rightarrow b_1=\frac{1}{2}, b_2=-\frac{7}{4}$

$$y(t)=-\frac{7}{4}\cos(2t)+\frac{1}{2}\sin(2t)+\frac{3}{4}$$

(b) (5%) $x(t)=e^{-t}$

$$y^p(t)=pe^{-t}$$
$$pe^{-t}+4pe^{-t}=-3e^{-t} \Rightarrow p=-\frac{3}{5}$$

$$\therefore y^p(t)=-\frac{3}{5}e^{-t}$$

$$y(t)=y^h(t)+y^p(t)=c_1e^{j2t}+c_2e^{-j2t}-\frac{3}{5}e^{-t}$$

$$\therefore y(t)=b_1\sin(2t)+b_2\cos(2t)-\frac{3}{5}e^{-t}$$

From $y(0^-)=-1, \frac{d}{dt}y(t)|_{t=0^-}=1 \Rightarrow b_1=\frac{1}{5}, b_2=-\frac{2}{5}$

$$\therefore y(t)=-\frac{2}{5}\cos(2t)+\frac{1}{5}\sin(2t)-\frac{3}{5}e^{-t}$$

(c) (10%) $x(t)=\sin(t)+\cos(t)$

$$y^p(t) = p_1\cos(t) + p_2\sin(t)$$

$$y^{'p}(t) = -p_1\sin(t) + p_2\cos(t)\ , y^{''p}(t) = -p_1\cos(t) - p_2\sin(t)$$

We get $p_1 = 1, p_2 = -1$

$$\therefore y^p(t) = \cos(t) - \sin(t)$$

$$\therefore y(t) = b1\cos(2t) + b2\sin(2t) + \cos(t) - \sin(t)$$

From $y(0^-) = -1, \frac{d}{dt}y(t)|_{t=0^-} = 1 \Rightarrow b_1 = -2, b_2 = 1$

$$\therefore y(t) = -2\cos(2t) + \sin(2t) + \cos(t) - \sin(t)$$

2. (20%)

$x[n] = u[n] \Rightarrow y[n] = s[n]$

(a)Homogeneous solution: $r^2 - 5r + 6 = 0 \Rightarrow r = 2,\ 3.$

Hence, $y^{(h)}(n) = c_1(2)^n + c_2(3)^n$.

Particular solution: Set $y^{(p)}[n] = Au[n]$.

$$\because\ y^{(p)}[n] - 5y^{(p)}[n-1] + 6y^{(p)}[n-2] = u[n] + u[n-1]$$

$$A - 5A + 6A = 1 + 1 = 2 \Rightarrow\ A = 1\ \Rightarrow\ \therefore \mathrm{y}^{(p)}[n] = u[n].$$

Complete solution:

$$y[n] = y^{(h)}[n] + y^{(p)}[n] = c_1(2)^n + c_2(3)^n + u[n].$$

$$y[-1] = y[2] = 0 \Rightarrow\ \mathrm{y}[0] = 1,\ \mathrm{y}[1] = 7.$$

$$\begin{cases} c_1 + c_2 = 0 \\ 2c_1 + 3c_2 = 6 \end{cases} \Rightarrow\ \mathrm{c}_1 = 6,\ \mathrm{c}_2 = -6.$$

$$\therefore y[n] = y^{(h)}[n] + y^{(p)}[n] = -6(2)^n + 6(3)^n + u[n].$$

(b)

Natural response:

$$y^{(n)}[n] = c_3(2)^n + c_4(3)^n,\ \mathrm{y}[-1] = y[-2] = 0.$$

$$c_3 = 0,\ c_4 = 0 \Rightarrow y^{(n)}[n] = 0.$$

(c)

Forced response:

$$y^{(f)}[n] = c_1(2)^n + c_2(3)^n + u[n],\ \mathrm{y}[-1] = y[-2] = 0.$$

$$\therefore\ y^{(f)}[n] = -6(2)^n + 6(3)^n + u[n].$$

3. (20%)

$$\frac{d}{dt}y(t) + 2y(t) = e^{3t}u(t)$$

Homogeneous solution：

$$r + 2 = 0 \Rightarrow r = -2$$
$$y^h(t) = c_1 e^{-2t}$$

Particular solution：

$$y^p(t) = pe^{3t}$$
$$3pe^{3t} + 2pe^{3t} = e^{3t}, t > 0$$
$$\Rightarrow p = \frac{1}{5}, t > 0$$
$$\therefore y^p(t) = \frac{1}{5}e^{3t}, t > 0$$

Complete solution：

$$y(t) = c_1 e^{-2t} + \frac{1}{5}e^{3t}, t > 0$$

Because at rest $\Rightarrow \therefore y(0) = 0$

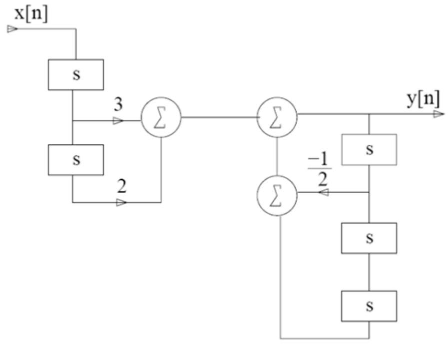
From $y(0) = 0 \Rightarrow c_1 = -\frac{1}{5}$

$$y(t) = -\frac{1}{5}e^{-2t} + \frac{1}{5}e^{3t}, t > 0$$

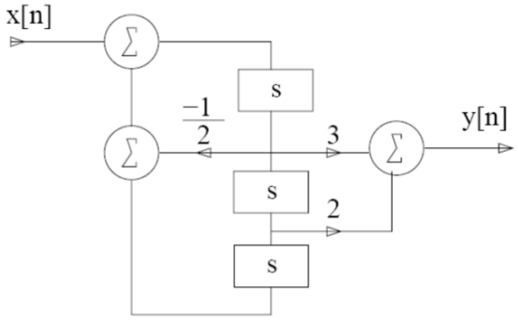
or $\Rightarrow y(t) = \frac{1}{5}(-e^{-2t} + e^{3t})u(t)$

4. (a) (10%)

Direct form I:

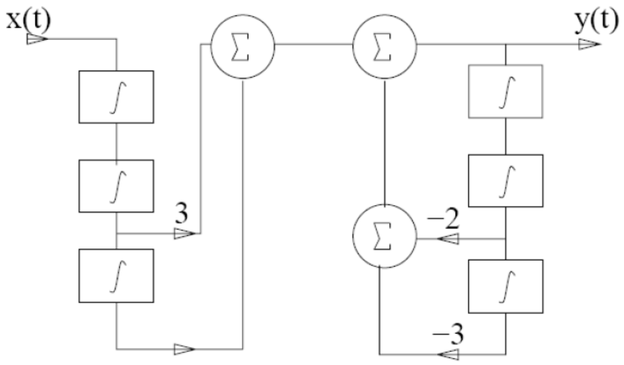


Direct form II:

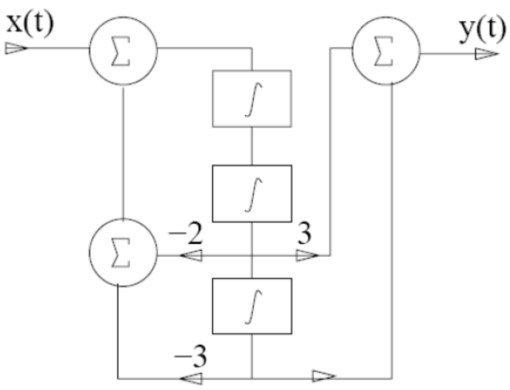



(b) (10%)

Direct form I:




Direct form II:




5. (20%)

(a)

$$f[n]=x[n]-\frac{1}{8}y[n]$$

$$y[n]=\frac{1}{2}x[n-1]+f[n-2]=\frac{1}{2}x[n-1]+x[n-2]-\frac{1}{8}y[n-2]$$

$$\Rightarrow y[n]+\frac{1}{8}y[n-2]=\frac{1}{2}x[n-1]+x[n-2]$$

(b)

$$\frac{d^2}{dt^2}y(t) = \frac{d}{dt}x(t) + 2\frac{d}{dt}y(t) - y(t)$$

$$\frac{d^2}{dt^2}y(t) - 2\frac{d}{dt}y(t) + y(t) = \frac{d}{dt}x(t).$$