Homework No. 2 Solution

1. (20%)

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k] \cdot h[n-k] = \sum_{k=-\infty}^{\infty} u[k-4] \cdot h[n-k]$$

$$y[n] = \sum_{k=4}^{\infty} h[n-k]$$

Evaluation the above summation:

For $n < 4$: $y[n] = 0$

For $n = 4$: $y[n] = h[0] = 1$

For $n = 5$: $y[n] = h[1] + h[0] = 2$

For $n = 6$: $y[n] = h[2] + h[1] + h[0] = 3$

For $n = 7$: $y[n] = h[3] + h[2] + h[1] + h[0] = 4$

For $n = 8$: $y[n] = h[4] + h[3] + h[2] + h[1] + h[0] = 2$

For $n \geq 9$: $y[n] = h[5] + h[4] + h[3] + h[2] + h[1] + h[0] = 0$

2.

(1) (30%) $x[n] = (-1)^n \left(u[n] - u[n-5]\right)$ and $h[n] = u[n+2]$.

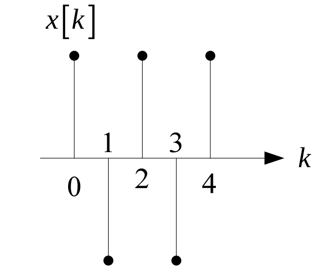
$n+2 < 0,\ n < -2,\ w_n[k] = 0,\ y[n] = 0$

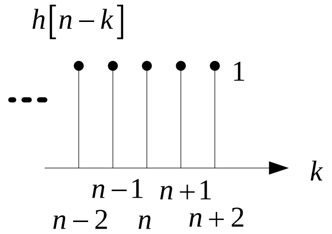
$0 \leq n+2 \leq 4,\ -2 \leq n \leq 2,\quad w_n[k] = (-1)^k,\ 0 \leq k \leq n+2$

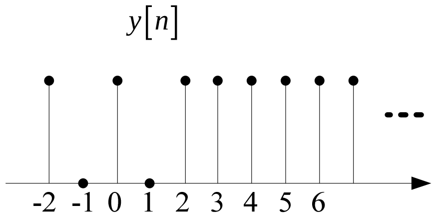
$$y[n] = \sum_{k=0}^{n+2} (-1)^k = \begin{cases} 1,\ n \text{ is odd} \\ 0,\ n \text{ is even} \end{cases}$$

$4 < n+2,\ 2 < n,\quad w_n[k] = (-1)^k,\ 0 \leq k \leq 4$

$$y[n] = \sum_{k=0}^{4} (-1)^k = 1$$







(2) (30%) $x[n]=u[n]-u[-n]$ and $h[n]=\begin{cases}\left(\frac{1}{2}\right)^n & , n\geq 0\\ 4^n & , n<0\end{cases}$.

$$h[n]=\begin{cases}\left(\frac{1}{2}\right)^n & , n\geq 0\\ 4^n & , n<0\end{cases}=\left(\frac{1}{2}\right)^n u[n]+4^n u[-n-1]$$

$$y[n]=x[n]*h[n]=u[n]*h[n]-u[-n]*h[n]$$

$$u[n]*h[n]=\sum_{k=-\infty}^{n}h[k]$$

$n\geq 0$,

$$\sum_{k=-\infty}^{n}h[k]=\sum_{k=-\infty}^{-1}4^k+\sum_{k=0}^{n}\left(\frac{1}{2}\right)^k$$

$$=\left(4^{-1}+4^{-2}+\cdots\right)+\left[1+\frac{1}{2}+\cdots+\left(\frac{1}{2}\right)^n\right]$$

$$=\frac{1}{3}+2\left[1-\left(\frac{1}{2}\right)^{n+1}\right]=\frac{7}{3}-\left(\frac{1}{2}\right)^n$$

$n<0$,

$$\sum_{k=-\infty}^{n}h[k]=\sum_{k=-\infty}^{n}4^k=4^n+4^{n-1}+\cdots$$

$$=4^n\left(1+4^{-1}+\cdots\right)=\frac{4}{3}4^n$$

$$u[-n]*h[n]=\sum_{k=n}^{\infty}h[k]$$

$n \geq 0$,

$$\sum_{k=n}^{\infty} h[k] = \sum_{k=n}^{\infty} \left(\frac{1}{2}\right)^k$$

$$= \left(\frac{1}{2}\right)^n + \left(\frac{1}{2}\right)^{n+1} + \cdots = \left(\frac{1}{2}\right)^n \left(1 + \frac{1}{2} + \cdots\right) = 2\left(\frac{1}{2}\right)^n$$

$n < 0$,

$$\sum_{k=n}^{\infty} h[k] = \sum_{k=n}^{-1} 4^k + \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k$$

$$= 4^{-1} + 4^{-2} + \cdots + 4^n + \left(1 + \frac{1}{2} + \cdots\right)$$

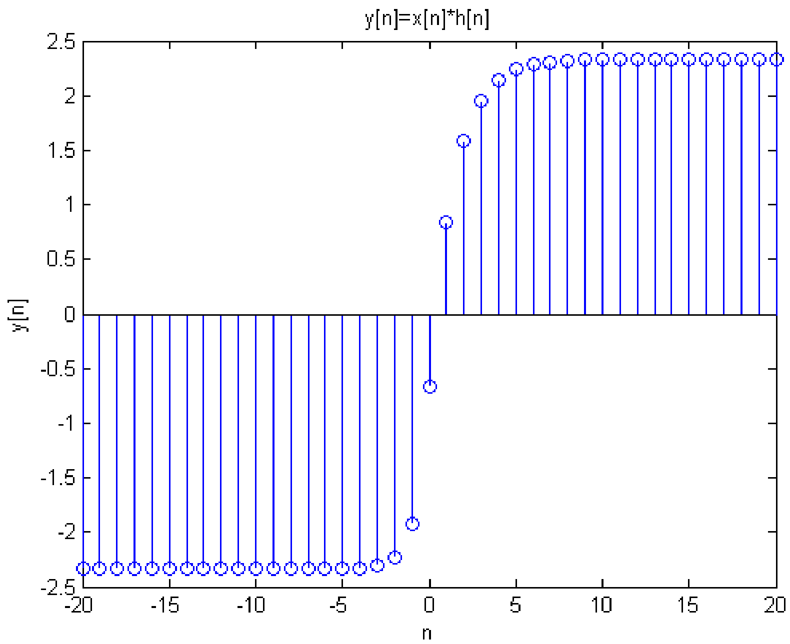
$$= 4^{-1}\left(1 + 4^{-1} + \cdots + 4^{n+1}\right) + 2$$

$$= 4^{-1} \times \frac{4}{3} \times \left(1 - 4^n\right) + 2 = \frac{1}{3}\left(1 - 4^n\right) + 2 = \frac{7}{3} - \frac{4^n}{3}$$

$$y[n] = \left[\frac{7}{3} - \left(\frac{1}{2}\right)^n\right] u[n] + \frac{4}{3} 4^n u[-n-1] - \left\{2\left(\frac{1}{2}\right)^n u[n] + \left(\frac{7}{3} - \frac{4^n}{3}\right) u[-n-1]\right\}$$

$$= \left[\frac{7}{3} - \left(\frac{1}{2}\right)^n - 2\left(\frac{1}{2}\right)^n\right] u[n] + \left\{\frac{4}{3} 4^n - \left(\frac{7}{3} - \frac{4^n}{3}\right)\right\} u[-n-1]$$

$$= \left[\frac{7}{3} - 3\left(\frac{1}{2}\right)^n\right] u[n] + \left(\frac{5}{3} 4^n - \frac{7}{3}\right) u[-n-1]$$



3. (20%)

$y(t) = 2t^2[u(t+1) - u(t-1)] * 2u(t+2).$

For $t+2 < -1,\ t < -3, y(t) = 0.$

For $t+2 < 1,\ -3 < t < -1,\ y(t) = 2\int_{-1}^{t+2} 2\tau^2 d\tau = \frac{4}{3}\tau^3\Big|_{-1}^{t+2} = \frac{4}{3}[(t+2)^3 + 1].$

For $t+2 \geq 1,\ -1 < t,\ y(t) = 2\int_{-1}^{1} 2\tau^2 d\tau = \frac{4}{3}\tau^3\Big|_{-1}^{1} = \frac{4}{3}[1+1] = \frac{8}{3}.$