Homework No. 1 Solution

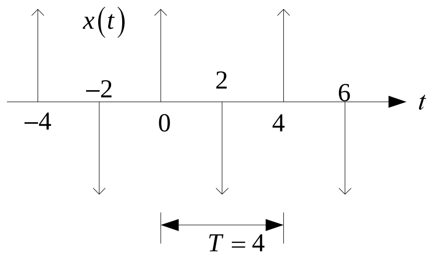
1. (18%)

(1) $x[n]=\cos\left(\frac{8}{15}\pi n\right)\Rightarrow N=\frac{2\pi m}{8\pi/15}=\frac{15m}{4}=15$ samples $(m=4)$, periodic

(2) $x(t)=\cos(2t)+\sin(3t)$

$$\left.\begin{array}{l} 2t=2\pi\frac{1}{T_1}t\Rightarrow T_1=\pi \\ 3t=2\pi\frac{1}{T_2}t\Rightarrow T_2=\frac{2\pi}{3}\end{array}\right\}\Rightarrow T_0=2\pi\text{ , periodic}$$

(3) $x(t)=\sum_{k=-\infty}^{\infty}(-1)^k\delta(t-2k)$



Periodic,

(4) Periodic (free points for this question due to *u*(0) is undefined)

Assume *u*(0)=0.5, then:

$$\begin{aligned} x(t)&=v(t)+v(-t)\\ &=\cos(t)u(t)+\cos(-t)u(-t)\\ &=\cos(t)\left[u(t)+u(-t)\right]=\cos(t)\end{aligned}$$

$$t=2\pi\frac{1}{T}t\Rightarrow T=2\pi\text{ sec}$$

(5) Non-periodic

$$x(t)=v(t)+v(-t)=\sin(t)u(t)+\sin(-t)u(-t)=\sin(t)\left[u(t)-u(-t)\right]$$

(6) Periodic

$$x[n]=\cos\left(\frac{1}{5}\pi n\right)\sin\left(\frac{1}{3}\pi n\right)=\frac{1}{2}\left\{\sin\left(\frac{8}{15}\pi n\right)+\sin\left(\frac{2}{15}\pi n\right)\right\}$$

$$\left.\begin{array}{l} \frac{8}{15}\pi N=2\pi m\Rightarrow N=\frac{15}{4}m=15,30,\ldots \\ \frac{2}{15}\pi N=2\pi l\Rightarrow N=15l=15,30,\ldots\end{array}\right\}\Rightarrow N=15\text{ samples}$$

2. (12%)

$$y(t) = y_1(t) + y_2(t) - y_4(t)$$
$$= x_1(t)x_1(t-1) + |x_2(t)| - \cos(1 + 2x_3(t))$$
$$= x(t)x(t-1) + |x(t)| - \cos(1 + 2x(t))$$

$$H:\ \ y(t) = x(t)x(t-1) + |x(t)| - \cos(1 + 2x(t))$$

3. (25%)

	Memory-less	Stable	Causal	Linear	Time Invariant
$y(t) = \cos(x(t))$	○	○	○	×	○
$y[n] = 2x[n]u[n]$	○	○	○	○	×
$y[n] = \log_{10}(\lvert x[n]\rvert)$	○	×	○	×	○
$y(t) = \int_{-\infty}^{t/2} x(\tau)d\tau$	×	×	×	○	×
$y[n] = \sum_{k=-\infty}^{n} x[k+2]$	×	×	×	○	○

(1)

$$y_1(t) = \cos(\alpha x_1(t));\ y_2(t) = \cos(\beta x_2(t))$$
$$y_3(t) = \cos(\alpha x_1(t) + \beta x_2(t)) \qquad \text{, nonlinear}$$
$$\neq \cos(\alpha x_1(t)) + \cos(\beta x_2(t)) = y_1(t) + y_2(t)$$

(2)

$$y_1[n-n_0] = 2x_1[n-n_0]u[n-n_0]$$
$$x_2[n] = x_1[n-n_0] \qquad \text{, time-varying}$$
$$y_2[n] = 2x_2[n]u[n] = 2x_1[n-n_0]u[n] \neq y_1[n-n_0]$$

(3)

- $x[n] = 0,\ \ |y[n]| = |\log_{10}(0)| = \infty$, unstable

- $$y_1[n] = \log_{10}\left(\left|\alpha x_1[n]\right|\right);\ y_2[n] = \log_{10}\left(\left|\beta x_2[n]\right|\right)$$ $$y_3[n] = \log_{10}\left(\left|\alpha x_1[n] + \beta x_2[n]\right|\right) \neq y_1[n] + y_2[n]$$, nonlinear

(4)

- Since the integrated range starts from negative infinite, the system has memory.
- $$\left|y(t)\right| = \left|\int_{-\infty}^{t/2} x(\tau)d\tau\right| \le \int_{-\infty}^{t/2} \left|x(\tau)\right| d\tau$$ $$= M_x \int_{-\infty}^{t/2} 1 d\tau = M_x\left(\frac{t}{2} + \infty\right) = \infty$$, unstable
- If $t < 0$, then y(t) is noncausal due to $t < 0.5t$.
- $$y_1(t - t_0) = \int_{-\infty}^{(t-t_0)/2} x_1(\tau)d\tau$$ $$x_2(t) = x_1(t - t_0)$$, time-varying $$y_2(t) = \int_{-\infty}^{t/2} x_2(\tau)d\tau = \int_{-\infty}^{t/2} x_1(\tau - t_0)d\tau$$ $$\overset{\tau' = \tau - t_0}{=} \int_{-\infty}^{t/2 - t_0} x_1(\tau')d\tau' \neq y_1(t - t_0)$$

(5)

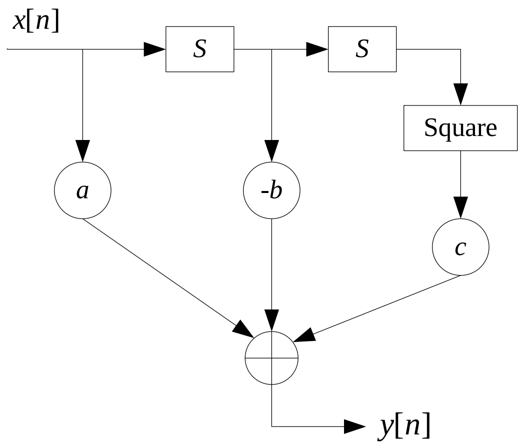
- $y[n] = \sum_{k=-\infty}^{n} x[k+2] = \cdots + x[n] + x[n+1] + x[n+2]$,

  memory and noncausal
- $\left|y[n]\right| = \left|\sum_{k=-\infty}^{n} x[k+2]\right| \le \sum_{k=-\infty}^{n} \left|x[k+2]\right| \le M_x(n+\infty) = \infty$, unstable

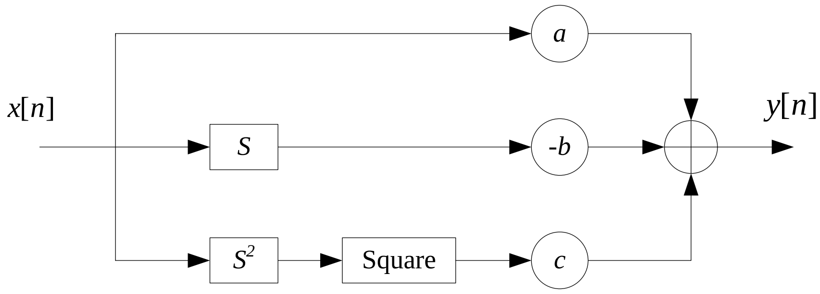
4. (10%)

$$y[n] = ax[n] - bx[n-1] + cx^2[n-2] = \left(a - bS + cS^2\right)\{x[n]\}$$

Cascade implementation of operator H:



Parallel implementation of operator *H*:



5.

(1) (10%)

$$\int_{-a}^{a} x(t)\,dt = \int_{-a}^{0} x(t)\,dt + \int_{0}^{a} x(t)\,dt$$
$$= \underbrace{\int_{a}^{0} x(-\lambda)\,d(-\lambda)}_{t=-\lambda} + \int_{0}^{a} x(t)\,dt = \int_{0}^{a} x(-\lambda)\,d\lambda + \int_{0}^{a} x(t)\,dt$$
$$= \underbrace{\int_{0}^{a} x(\lambda)\,d\lambda}_{even,\ x(-\lambda)=x(\lambda)} + \int_{0}^{a} x(t)\,dt = 2\int_{0}^{a} x(t)\,dt$$

$$\sum_{n=-k}^{k} x[n] = \sum_{n=-k}^{-1} x[n] + x[0] + \sum_{n=1}^{k} x[n]$$
$$= \underbrace{\sum_{m=k}^{1} x[-m]}_{n=-m} + x[0] + \sum_{n=1}^{k} x[n]$$
$$= \underbrace{\sum_{m=1}^{k} x[m]}_{even,\ x[-m]=x[m]} + x[0] + \sum_{n=1}^{k} x[n] = x[0] + 2\sum_{n=1}^{k} x[n]$$

(2) (10%)

Since $x(t)$ and $x[n]$ are odd, that is, $x(t) = -x(-t)$ and $x[n] = -x[-n]$, we have

$$x(0) = -x(-0) \text{ and } x[0] = -x[-0]$$

Hence,

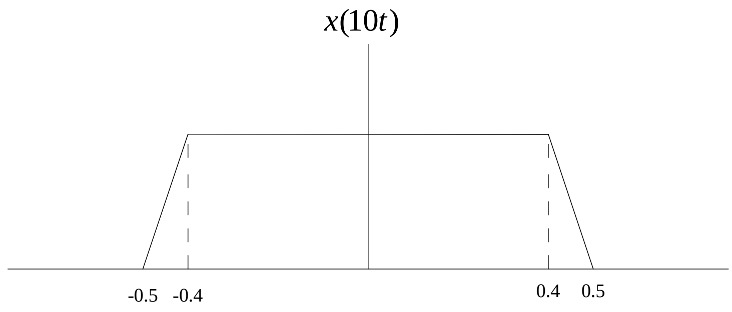
$$x(0) = -x(-0) = -x(0) \Rightarrow x(0) = 0$$
$$x[0] = -x[-0] = -x[0] \Rightarrow x[0] = 0$$

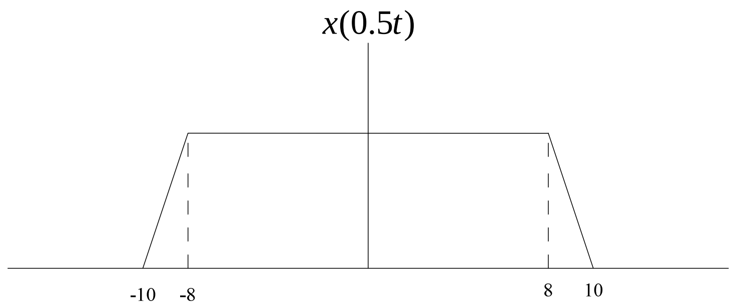
$$\int_{-a}^{a} x(t)\,dt = \int_{-a}^{0} x(t)\,dt + \int_{0}^{a} x(t)\,dt$$
$$= \underbrace{\int_{a}^{0} x(-\lambda)\,d(-\lambda)}_{t=-\lambda} + \int_{0}^{a} x(t)\,dt = \underbrace{\int_{0}^{a} -x(\lambda)\,d\lambda}_{odd,\ -x(-\lambda)=x(\lambda)} + \int_{0}^{a} x(t)\,dt = 0$$

$$\sum_{n=-k}^{k} x[n] = \sum_{n=-k}^{-1} x[n] + x[0] + \sum_{n=1}^{k} x[n]$$
$$= \underbrace{\sum_{m=k}^{1} x[-m]}_{n=-m} + x[0] + \sum_{n=1}^{k} x[n]$$
$$= \underbrace{\sum_{m=1}^{k} -x[m]}_{odd,\ -x[-m]=x[m]} + x[0] + \sum_{n=1}^{k} x[n] = x[0] = 0$$

6.

(1) (10%)





(2) (5%)