# Homework 7 Solution

1.  $X(s) = \dfrac{2s}{s^2 - 4} = \dfrac{1}{s+2} + \dfrac{1}{s-2}$

    (1) Stable:  $x(t) = e^{-2t}u(t) - e^{2t}u(-t)$, ROC: $-2 < \mathrm{Re}(s) < 2$.  (10%)

    (2) Causal:  $x(t) = e^{-2t}u(t) + e^{2t}u(t)$, ROC: $2 < \mathrm{Re}(s)$.  (10%)

    (3) Anti-causal:  $x(t) = -e^{-2t}u(-t) - e^{2t}u(-t)$, ROC: $\mathrm{Re}(s) < -2$.  (10%)

    (4) Because $X(s)$ has a pole in the right half of the $s$-plane. (10%)

2.

    (1) (5%)

    $$sX(s) + X(s) \xleftrightarrow{\;\mathcal{L}_u\;} \frac{dx(t)}{dt} + x(t) = -2\sin(2t)u(t) + \delta(t) + \cos(2t)u(t)$$

    (2) (5%)

    $$X(s+2) \xleftarrow{\;\mathcal{L}_u\;} e^{-2t}x(t)$$
    $$x(t) \quad = \quad e^{-2t}\cos(2t)u(t)$$

    (3) (5%)

    $$B(s) = \frac{1}{s}X(s) \xleftarrow{\;\mathcal{L}_u\;} \int_{-\infty}^{t} x(\tau)\,d\tau$$
    $$\xleftarrow{\;\mathcal{L}_u\;} \int_{-\infty}^{t} \cos(2\tau)u(\tau)\,d\tau$$
    $$\xleftarrow{\;\mathcal{L}_u\;} \int_{0}^{t} \cos(2\tau)\,d\tau$$

    $$B(s) \xrightarrow{\;\mathcal{L}_u\;} \frac{1}{2}\sin(2t)$$
    $$\frac{1}{s}B(s) \xrightarrow{\;\mathcal{L}_u\;} \int_{0}^{t} \frac{1}{2}\sin(2\tau)\,d\tau$$
    $$\xleftarrow{\;\mathcal{L}_u\;} \frac{1 - \cos(2t)}{4}u(t)$$

    (4) (5%)

    $$A(s) = e^{-3s}X(s) \xrightarrow{\;\mathcal{L}_u\;} a(t) = x(t-3) = \cos(2(t-3))u(t-3)$$
    $$B(s) = \frac{d}{ds}A(s) \xrightarrow{\;\mathcal{L}_u\;} b(t) = -ta(t) = -t\cos(2(t-3))u(t-3)$$

3.  (20%)

Since $x(t)$ is real, the poles of $X(s)$ must occur in conjugate reciprocal pairs. From 1~3, we can know that $X(s)$ is of the form:

$$X(s) = \frac{A}{(s+1-j)(s+1+j)}.$$

From 5, we know that $X(0) = \dfrac{A}{(1-j)(1+j)} = 8$. Therefore, we may deduce that

A=16 and $X(s) = \dfrac{16}{s^2 + 2s + 2}$.
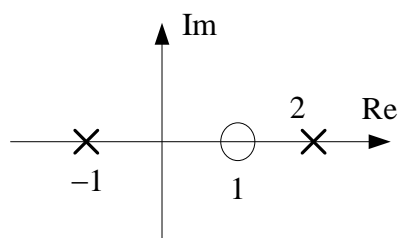
Let $R_x$ denote the ROC of $X(s)$. From the poles locations, we know that there are two possible choices of $R_x$. $R_x$ may either be $\text{Re}\{s\} < -1$ or $\text{Re}\{s\} > -1$. Note that

$$y(t) = e^{2t}x(t) \xleftrightarrow{\ \mathcal{L}\ } Y(s) = X(s-2).$$

The ROC of $Y(s)$ is $R_x$ shifted by 2 to the right. Since it is given that $y(t)$ is not absolutely integrable, the ROC of $Y(s)$ should not include the $j\omega - \text{axis}$. This is possible only if $R_x$ is $\text{Re}\{s\} > -1$.

4.  $\dfrac{d^2 y(t)}{dt^2} - \dfrac{dy(t)}{dt} - 2y(t) = \dfrac{dx(t)}{dt} - x(t).$

(1)  $H(s) = \dfrac{Y(s)}{X(s)} = \dfrac{s-1}{s^2 - s - 2} = \dfrac{s-1}{(s-2)(s+1)}.$



(2)