Homework 7 Solution

1. $X(s) = \frac{2s}{s^2-4} = \frac{1}{s+2} + \frac{1}{s-2}$

(1) Stable: $x(t) = e^{-2t}u(t) - e^{2t}u(-t)$, ROC: $-2 < \text{Re}(s) < 2$. (10%)

(2) Causal: $x(t) = e^{-2t}u(t) + e^{2t}u(t)$, ROC: $2 < \text{Re}(s)$. (10%)

(3) Anti-causal: $x(t) = -e^{-2t}u(-t) - e^{2t}u(-t)$, ROC: $\text{Re}(s) < -2$. (10%)

(4) Because $X(s)$ has a pole in the right half of the s-plane. (10%)

2.

(1) (5%)

$$\begin{aligned} sX(s) + X(s) &\xleftrightarrow{\mathcal{L}_u} \frac{d}{dt}x(t) + x(t) \\ &= [-2\sin(2t) + \cos(2t)]\, u(t) \end{aligned}$$

(2) (5%)

$$\begin{aligned} X(s+2) &\xleftrightarrow{\mathcal{L}_u} e^{-2t}x(t) \\ x(t) &= e^{-2t}\cos(2t)u(t) \end{aligned}$$

(3) (5%)

$$\begin{aligned} B(s) = \frac{1}{s}X(s) &\xleftrightarrow{\mathcal{L}_u} \int_{-\infty}^{t} x(\tau)d\tau \\ &\xleftrightarrow{\mathcal{L}_u} \int_{-\infty}^{t} \cos(2\tau)u(\tau)d\tau \\ &\xleftrightarrow{\mathcal{L}_u} \int_{0}^{t} \cos(2\tau)d\tau \end{aligned}$$

$$\begin{aligned} B(s) &\xleftrightarrow{\mathcal{L}_u} \frac{1}{2}\sin(2t) \\ \frac{1}{s}B(s) &\xleftrightarrow{\mathcal{L}_u} \int_0^t \frac{1}{2}\sin(2\tau)d\tau \\ &\xleftrightarrow{\mathcal{L}_u} \frac{1-\cos(2t)}{4}u(t) \end{aligned}$$

(4) (5%)

$$\begin{aligned} A(s) = e^{-3s}X(s) &\xleftrightarrow{\mathcal{L}_u} a(t) = x(t-3) = \cos(2(t-3))u(t-3) \\ B(s) = \frac{d}{ds}A(s) &\xleftrightarrow{\mathcal{L}_u} b(t) = -ta(t) = -t\cos(2(t-3))u(t-3) \end{aligned}$$

3. (20%)

Since $x(t)$ is real, the poles of $X(s)$ must occur in conjugate reciprocal pairs. From 1~3, we can know that $X(s)$ is of the form:

$$X(s) = \frac{A}{(s+1-j)(s+1+j)}.$$

From 5, we know that $X(0) = \frac{A}{(1-j)(1+j)} = 8$. Therefore, we may deduce that A=16 and $X(s) = \frac{16}{s^2+2s+2}$.

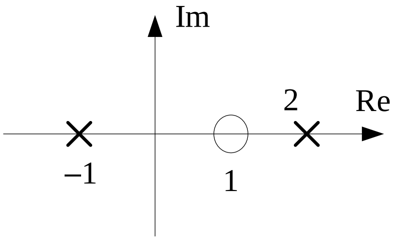
Let R_x denote the ROC of $X(s)$. From the poles locations, we know that there are two possible choices of R_x. R_x may either be Re{s} < -1 or Re{s} > -1. Note that

$$y(t) = e^{2t}x(t) \overset{\mathcal{L}}{\longleftrightarrow} Y(s) = X(s-2).$$

The ROC of $Y(s)$ is R_x shifted by 2 to the right. Since it is given that $y(t)$ is not absolutely integrable, the ROC of $Y(s)$ should not include the $j\omega-$axis. This is possible only if R_x is Re{s} > -1.

4. $\frac{d^2y(t)}{dt^2} - \frac{dy(t)}{dt} - 2y(t) = \frac{dx(t)}{dt} - x(t).$

(1) $H(s) = \frac{Y(s)}{X(s)} = \frac{s-1}{s^2-s-2} = \frac{s-1}{(s-2)(s+1)}.$



(2)