Homework 6 Solution

1. (55%)

(1) $x(t)=e^{-t}u(t+4)$

$$X(s)=\int_{-\infty}^{\infty}e^{-t}u(t+4)e^{-st}dt=\int_{-4}^{\infty}e^{-t}e^{-st}dt$$

$$=\int_{-4}^{\infty}e^{-t(1+s)}dt=\left.\frac{-e^{-t(1+s)}}{1+s}\right|_{-4}^{\infty}=\frac{e^{4(1+s)}}{1+s},\ \ \text{Re}\{s+1\}>0\Rightarrow \text{ROC}:\text{Re}\{s\}>-1$$

(2) $x(t)=\sin(t)u(t)$

$$X(s)=\int_{0}^{\infty}\frac{1}{2j}\left(e^{jt}-e^{-jt}\right)e^{-st}dt=\int_{0}^{\infty}\frac{1}{2j}e^{t(j-s)}dt-\int_{0}^{\infty}\frac{1}{2j}e^{-t(j+s)}dt$$

$$=\frac{1}{2j}\left(\frac{-1}{j-s}-\frac{1}{j+s}\right)=\frac{1}{1+s^2}$$

$$\text{Re}\{j-s\}<0 \text{ and } \text{Re}\{j+s\}>0\Rightarrow \text{ROC}:\text{Re}\{s\}>0$$

2. $X(s)=\dfrac{-s-4}{s^2+3s+2}=\dfrac{-3}{s+1}+\dfrac{2}{s+2}$

(1) With ROC $\text{Re}\{s\}<-2$

Left-sided: $x(t)=\left(3e^{-t}-2e^{-2t}\right)u(-t)$. (15%)

(2) With ROC $\text{Re}\{s\}>-1$

Right-sided: $x(t)=\left(-3e^{-t}+2e^{-2t}\right)u(t)$. (15%)

(3) With ROC $-2<\text{Re}\{s\}<-1$

Two-sided: $x(t)=3e^{-t}u(-t)+2e^{-2t}u(t)$. (15%)