Homework 2

Due 18:10, 22 Oct 2009

1. Find and sketch $y[n] = x[n] * h[n]$ as $x[n] = (-1)^n (u[n] - u[n-7])$ and $h[n] = u[n+3]$.

2. Evaluate the following continuous-time convolution integral:
   $y(t) = \cos(\pi t)[u(t+1) - u(t-1)] * [u(t+1) - u(t-1)]$.

3. Consider the difference equation

$$y[n] - \frac{1}{2} y[n-1] = x[n] \qquad (1)$$

and suppose that

$$x[n] = (\frac{1}{3})^n u[n]. \qquad (2)$$

Assume that the solution $y[n]$ consists of the sum of a particular solution $y_p[n]$ to the first equation and homogeneous solution $y_h[n]$ satisfying the equation

$$y_h[n] - \frac{1}{2} y_h[n-1] = 0.$$

(i) Verify the homogeneous solution given by

$$y_h[n] = A(\frac{1}{2})^n.$$

(ii) Consider the particular solution

$$y_p[n] = \frac{1}{2} y_p[n-1] = (\frac{1}{3})^n u[n].$$

Assuming that, $y_p[n]$ is in the form of $B(\frac{1}{3})^n$ for $n \ge 0$, and substitution this in the above difference equation, determine the value B.

(iii) Suppose that the LTI system described in equation (1) and initially at rest as the input signal specified in equation (2). Since x[n]=0 for n<0, we have y[n]=0 for n<0. Also from part (i) and (ii), we have:

$$y[n] = A(\frac{1}{2})^n + B(\frac{1}{3})^n \quad \text{for} \quad n \ge 0.$$

In order to solve the unknown constant A, to specify a value for y[n] as $n \ge 0$ is required. Use the condition of initial rest and equations above to determine y[0] and A.

4. Consider a system with input x(t) and output y(t) satisfy the first-order differential equation

$$\frac{dy(t)}{dt} + 2y(t) = x(t).$$

The system also satisfies the condition of initial rest, determine the system output y(t) as the input is $x(t) = e^{3t}u(t)$.

5. Determine the homogeneous and particular solutions for the system described by the following differential equation for the given inputs and initial conditions:

$$\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} + y(t) = \frac{dx(t)}{dt}$$

$$y(0^-) = -1, \left.\frac{dy(t)}{dt}\right|_{t=0^-} = 1.$$

(i) x(t)=t

(ii) x(t)=sin(t)+cos(t)

6. Identify the natural and forced responses of the system described by the following difference equation with the input x[n] and initial conditions as:

$$y[n] - \frac{1}{2}y[n-1] = 2x[n]$$

$$y[-1] = 3$$

$$x[n] = (\frac{-1}{2})^n u[n]$$