

## Homework No. 1

**Due 18:10, March 12, 2009**

**1.** Determine whether the following signals are periodic, and for those which are, find the fundamental period: (18%)

(1)  $x[n] = \cos\left(\frac{8}{15}\pi n\right)$ ; (2)  $x(t) = \cos(2t) + \sin(3t)$ ;

(3)  $x(t) = \sum_{k=-\infty}^{\infty} (-1)^k \delta(t - 2k)$ ; (4)  $x(t) = v(t) + v(-t)$ , where  $v(t) = \cos(t)u(t)$ ;

(5)  $x(t) = v(t) + v(-t)$ , where  $v(t) = \sin(t)u(t)$ ; (6)  $x[n] = \cos\left(\frac{1}{5}\pi n\right)\sin\left(\frac{1}{3}\pi n\right)$ .

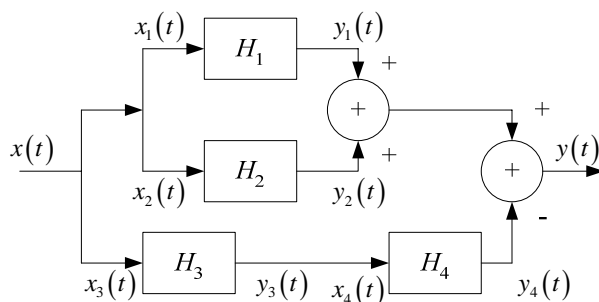
**2.** A system consists of several subsystems connected as shown in Fig. 1. Find the operator  $H$  relating  $x(t)$  to  $y(t)$  for the following subsystem operators: (12%)

$$H_1 : y_1(t) = x_1(t)x_1(t-1);$$

$$H_2 : y_2(t) = |x_2(t)|;$$

$$H_3 : y_3(t) = 1 + 2x_3(t);$$

$$H_4 : y_4(t) = \cos(x_4(t)).$$



**Figure 1**

**3.** The system that follow have input  $x(t)$  or  $x[n]$  and output  $y(t)$  or  $y[n]$ . For each system, determine whether it is (i) memoryless, (ii) stable, (iii) causal, (iv) linear, and (v) time invariant. (25%)

(1)  $y(t) = \cos(x(t))$ ; (2)  $y[n] = 2x[n]u[n]$ ; (3)  $y[n] = \log_{10}(|x[n]|)$ ;

(4)  $y(t) = \int_{-\infty}^{t/2} x(\tau)d\tau$ ; (5)  $y[n] = \sum_{k=-\infty}^n x[k+2]$ .

4. The output of a discrete-time system is related to its input  $x[n]$  as follows:

$$y[n] = a \cdot x[n] - b \cdot x[n-1] + c \cdot x^2[n-2]$$

Let the operator  $S^k$  denote a system that shifts the input  $x[n]$  by  $k$  time units to produce  $x[n-k]$ . Formulate the operator  $H$  for the system relating  $y[n]$  to  $x[n]$ . Then develop a block diagram representation for  $H$ , using (a) cascade implementation and (b) parallel implementation. (10%)

5. Show that

(1) If  $x(t)$  and  $x[n]$  are even, then

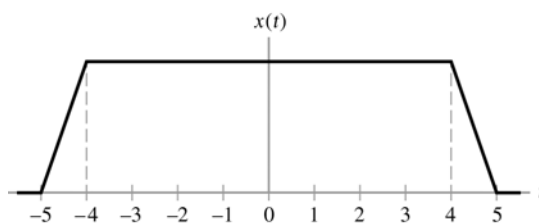
$$\int_{-a}^a x(t) dt = 2 \int_0^a x(t) dt; \quad \sum_{n=-k}^k x[n] = x[0] + 2 \sum_{n=1}^k x[n] \quad (10\%)$$

(2) If  $x(t)$  and  $x[n]$  are odd, then

$$x(0) = 0 \text{ and } x[n] = 0; \quad \int_{-a}^a x(t) dt = 0 \quad \text{and} \quad \sum_{n=-k}^k x[n] = 0 \quad (10\%)$$

6 (15%)

(1) The trapezoidal pulse  $x(t)$  of Fig. 2 is time scaled, producing the equation  $y(t) = x(at)$ . Sketch  $y(t)$  for  $a = 10$  and  $0.5$ .



**Figure 2**

(2) Sketch the trapezoidal pulse  $y(t)$  related to that of Fig. 2 as follows

$$y(t) = x(5(t-1))$$