Homework No. 8 Solution

1.

(1)

$$X(z)=\frac{(1/35)}{(1+\frac{1}{2}z^{-2})^2}+\frac{(58/1225)}{(1+\frac{1}{2}z^{-1})}-\frac{(1568/1225)}{(1-2z^{-1})}+\frac{(2700/1225)}{(1-3z^{-1})}$$

$$x[n]=\frac{1}{35}(n+1)(\frac{-1}{2})^{n+1}u[n+1]+\frac{58}{1225}(\frac{-1}{2})^n u[n]+\frac{1568}{1225}(2)^n u[-n-1]-\frac{2700}{1225}(3)^n u[-n-1]$$

(2)

$$X(z)=e^{z^{-1}}=1+z^{-1}+\frac{z^{-2}}{2!}+\frac{z^{-3}}{3!}\ldots$$

$$x[n]=\frac{1}{n!}u[n]$$

(3)

$$X(z)=z^2+2z+\frac{2}{1-2z^{-1}},|z|<2$$

$$x[n]=\delta[n+2]+2\delta[n+1]-2(2)^n u[-n-1]$$

2.

(1) 1/2 < |z| < 2

(2) Two-sided

(3) |z|<3/4

(4) Causal

(5) 0

(6) $H(z)=\frac{z^{-1}(1+\frac{3}{4}z^{-1})}{(1-2z^{-1})}$, |z|<2

(7) anticausal

3. x[n] is real, poles and zeros occur in conjugate pairs.

Assume X(z) has 2 or less zeros. If X(z) has more than 2 zeros. Than X(z) must have pole at infinity.

X(z) has 2 zeros at the origin.

$$X(z)=\frac{Az^2}{(z-\frac{1}{2}e^{j\pi/3})(z-\frac{1}{2}e^{-j\pi/3})}$$

X(1)=8/3, so A=2

4.

(1)

$$H(z) = \frac{z^{-1}}{1 - z^{-1} - z^{-2}}$$

poles : $z = (1/2) \pm (\sqrt{5}/2)$

zero : $z = 0$

$h[n]$ is causal

ROC: $|z| > (1/2) + (\sqrt{5}/2)$

(2)

ROC : $(\sqrt{5}/2) - (1/2) < |z| < (1/2) + (\sqrt{5}/2)$

$$H(z) = -\frac{1/\sqrt{5}}{1 - (\frac{1+\sqrt{5}}{2})z^{-1}} + \frac{1/\sqrt{5}}{1 - (\frac{1-\sqrt{5}}{2})z^{-1}}$$

$$h[n] = \frac{1}{\sqrt{5}}(\tfrac{1+\sqrt{5}}{2})^n u[-n-1] + \frac{1}{\sqrt{5}}(\tfrac{1-\sqrt{5}}{2})^n u[n]$$