Homework No. 1

Due 10:10 am, March 22, 2007

1. Show that

(1) If $x(t)$ and $x[n]$ are even, then

$$\int_{-a}^{a} x(t)\,dt = 2\int_{0}^{a} x(t)\,dt\,;\quad \sum_{n=-k}^{k} x[n] = x[0] + 2\sum_{n=1}^{k} x[n]$$

(2) If $x(t)$ and $x[n]$ are odd, then

$$x(0) = 0 \text{ and } x[n] = 0;\quad \int_{-a}^{a} x(t)\,dt = 0 \quad\text{and}\quad \sum_{n=-k}^{k} x[n] = 0$$

2. Determine whether the following signals are periodic, and for those which are, find the fundamental period:

(1) $x[n] = \cos(\frac{8}{15}\pi n)$; (2) $x(t) = \cos(2t) + \sin(3t)$;

(3) $x(t) = \sum_{k=-\infty}^{\infty} (-1)^k \delta(t-2k)$; (4) $x(t) = v(t) + v(-t)$, where $v(t) = \cos(t)u(t)$;

(5) $x(t) = v(t) + v(-t)$, where $v(t) = \sin(t)u(t)$; (6) $x[n] = \cos(\frac{1}{5}\pi n)\sin(\frac{1}{3}\pi n)$.

3.

(1) The trapezoidal pulse $x(t)$ of Fig. 1 is time scaled, producing the equation $y(t) = x(at)$. Sketch $y(t)$ for $a = 5$ and 0.2.

(2) Sketch the trapezoidal pulse $y(t)$ related to that of Fig. 1 as follows

$$y(t) = x(10t - 5)$$

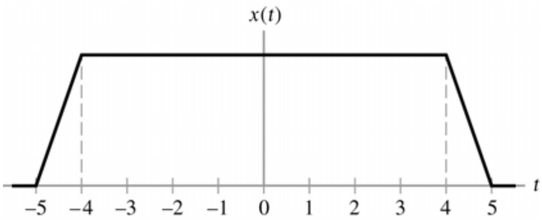

Figure 1

4.

(1) Show that if $x(t)$ is periodic with period T_0, then the time-averaged power P of $x(t)$ is the same as the average power of $x(t)$ over any interval of length T_0. That is, $P = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x^2(t)\,dt$.

(2) Determine whether the following signals are energy signals, power signals, or neither.

(a) $x(t) = e^{-at}u(t),\ a > 0$; (b) $x(t) = A\cos(\omega_0 t + \theta)$;

(c) $x(t) = tu(t)$; (d) $x[n] = (-0.5)^n u[n]$; (e) $x[n] = u[n]$.

5. Evaluate following integrals:

(1) $\int_{-\infty}^{\infty} \left(t^2 + \cos(\pi t)\right) \delta(t-1)\, dt$

(2) $\int_{-\infty}^{\infty} e^{-t} \delta(2t-2)\, dt$

6. Consider the discrete-time signal

$$x[n] = 1 - \sum_{k=3}^{\infty} \delta[n-1-k]$$

Determine the values of the integers M and n_0 so that $x[n]$ may be expressed as $x[n] = u[Mn - n_0]$.

7. Consider three systems with the following input-output relationships:

$$H_1 : y[n] = \begin{cases} x\{n/2\}, & n \text{ even} \\ 0, & n \text{ odd} \end{cases},$$

$$H_2 : y[n] = x[n] + \frac{1}{2}x[n-1] + \frac{1}{4}x[n-2],$$

$$H_3 : y[n] = x[2n].$$

Suppose that these systems are connected in series as depicted in Fig. 1. Find the input-output relationship for the overall interconnected system. Is this system linear? Is it time invariant?

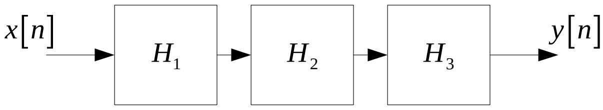

Figure 1

8. The system that follow have input $x(t)$ or $x[n]$ and output $y(t)$ or $y[n]$. For each system, determine whether it is (i) memoryless, (ii) stable, (iii) causal, (iv) linear, and (v) time invariant. Justify your answers.

(1) $y[n] = \cos(2\pi x[n+1]) + x[n]$; (2) $y(t) = \frac{d}{dt}\left\{e^{-t} x(t)\right\}$;

(3) $y[n] = \log_{10}(|x[n]|)$; (4) $y(t) = \int_{-\infty}^{2t} x(\tau)\, d\tau$; (5) $y(t) = x(t/2)$;

(6) $y[n] = 2x[2^n]$.

9. The output of a discrete-time system is related to its input $x[n]$ as follows:

$$y[n] = a_0 x[n] + a_1 x[n-1] + a_2 x[n-2] + a_3 x[n-3]$$

(1) Show that the discrete-time system is BIBO stable for all a_0, a_1, a_2, and a_3.

(2) How far does the memory of the discrete-time system extend into the past?

10. For each of these following statements, determine whether it is true, and if so, determine the relationship between the fundamental periods of the two signals considered in the statement. If the statement is not true, produce a counterexample to it.

(1) Let $x(t)$ be a continuous-time signal, and let $y(t) = x(2t)$. Consider the following statements:

(a) If $x(t)$ is periodic, then $y(t)$ is periodic.

(b) If $y(t)$ is periodic, then $x(t)$ is periodic.

(2) Let $x[n]$ be a discrete-time signal, and let $y[n] = x[2n]$.

(a) If $x[n]$ is periodic, then $y[n]$ is periodic.

(b) If $y[n]$ is periodic, then $x[n]$ is periodic.