Homework No. 8

Due 10:10 am, June 6, 2006

1. For the following integral, specify the value of the real parameter σ which ensure that the integral converges: (5%)

$$\int_{-\infty}^{\infty} e^{-10|t|} e^{-(\sigma+j\omega)t} dt .$$

2. Determine the **bilateral** Laplace transform and ROC for the following signals:

(1) $x(t) = e^{-t}u(t+3)$ (7%) (2) $x(t) = \sin(t)u(t)$ (8%)

3. Use the tables of transforms and properties to determine the time signals that correspond to the following **bilateral** Laplace transforms:

(1) $X(s) = e^{5s}\dfrac{1}{s+3}$ with ROC $\mathrm{Re}\{s\} < -3$ (7%)

(2) $X(s) = s^{-1}\dfrac{d}{ds}\left(\dfrac{e^{-3s}}{s}\right)$ with ROC $\mathrm{Re}\{s\} > 0$ (8%)

4. Use the method of partial fractions to determine the time signal corresponding to the following **bilateral** Laplace transform:

$$X(s) = \frac{2s^2 + 2s - 2}{s^2 - 1}$$

(1) With ROC $\mathrm{Re}\{s\} < -1$ (5%)

(2) With ROC $\mathrm{Re}\{s\} > 1$ (5%)

(3) With ROC $-1 < \mathrm{Re}\{s\} < 1$ (5%)

5.

(1) A system has the indicated transfer function $H(s)$. Determine the impulse response, assuming (a) that the system is causal and (b) that the system is stable. (8%)

$$H(s) = \frac{2s-1}{s^2 + 2s + 1}$$

(2) A stable system has the indicated input $x(t)$ and output $y(t)$. Use Laplace transforms to determine the transfer function and impulse response of the system. (7%)

$$x(t) = e^{-2t}u(t),\ y(t) = -2e^{-t}u(t) + 2e^{-3t}u(t)$$

6. Determine the **unilateral** Laplace transform of the following signals, using the

<u>defining equation</u>:

(1) $x(t) = u(t) - u(t-10)$ (7%)

(2) $x(t) = \begin{cases} \sin(\pi t), & 0 < t < 1 \\ 0, & \text{otherwise} \end{cases}$ (8%)

7. Given the transform pair $x(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{2s}{s^2+2}$, where $x(t) = 0$ for $t < 0$, determine the Laplace transform of the following time signals: (20%)

(1) $x(t-3)$

(2) $x(3t)$

(3) $x(t) * \frac{d}{dt}x(t)$

(4) $e^{-2t}x(t)$

(5) $\int_0^t x(3\tau)\,d\tau$

Note: Please turn in your homework by 5:00pm, June 6.