Homework No. 3 Solution

1. $y[n]=\sum_{k=-\infty}^{\infty} x[k]g[n-2k]$. $g[n]=u[n]-u[n-4]$

(1) $x[n]=\delta[n-1]$.

$$y[n]=\sum_{k=-\infty}^{\infty}\delta[k-1]g[n-2k]=g[n-2]=u[n-2]-u[n-6]$$

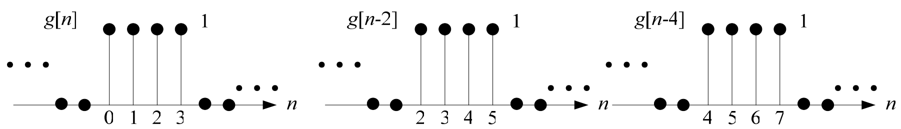
(2) $x[n]=\delta[n-2]$.

$$y[n]=\sum_{k=-\infty}^{\infty}\delta[k-2]g[n-2k]=g[n-4]=u[n-4]-u[n-8]$$

(3) If the system S is time invariant then the system output obtained in part (2) has to be the same as the system output obtained in part (1) shifted by 1 to the right. Clearly, this is not the case. Therefore, the system is not LTI.

(4) $x[n]=u[n]$

$$y[n]=\sum_{k=0}^{\infty} g[n-2k]=\begin{cases}1, & n=0,1\\ 2, & n>1\\ 0, & \text{otherwise}\end{cases}=2u[n]-\delta[n]-\delta[n-1]$$



2.

(1) $y[n]=(-1)^n * 2^n u[-n+2]$.

$$\begin{aligned}y[n]&=\sum_{k=-\infty}^{\infty}2^k u[-k+2](-1)^{n-k}\\&=\sum_{k=-\infty}^{2}2^k(-1)^{n-k}=(-1)^n\sum_{k=-\infty}^{2}2^k(-1)^{-k}\\&=(-1)^n\sum_{k=-\infty}^{2}(-2)^k\\&=(-1)^n\left[(-2)^2+(-2)^1+1+(-2)^{-1}+(-2)^{-2}+\cdots\right]\\&=(-1)^n\left[4+(-2)^1+\frac{1}{1-(-2)^{-1}}\right]=\frac{8}{3}(-1)^n\end{aligned}$$

(2) $y[n]=\left(u[n+10]-2u[n]+u[n-4]\right)*\beta^n u[n],\ |\beta|<1$.

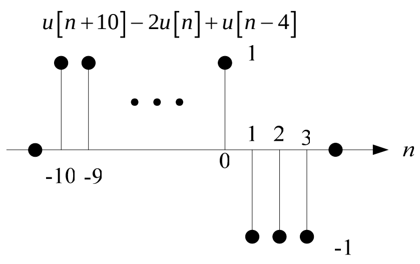
For $n<-10$, $y[n]=0$.

For $n<0$, $y[n]=\sum_{k=-10}^{n}\beta^{n-k}=\beta^n\sum_{k=-10}^{n}\beta^{-k}=\dfrac{\beta^{n+11}-1}{\beta-1}$

For $n \le 3$, $y[n] = \beta^n \sum_{k=-10}^{-1} \beta^{-k} - \beta^n \sum_{k=0}^{n} \beta^{-k} = \dfrac{\beta^{n+11} - \beta^{n+1}}{\beta - 1} - \dfrac{\beta^{n+1} - 1}{\beta - 1}$.

For $n > 3$,

$$y[n] = \beta^n \sum_{k=-10}^{-1} \beta^{-k} - \beta^n \sum_{k=0}^{3} \beta^{-k} = \frac{\beta^{n+11} - \beta^{n+1}}{\beta - 1} - \frac{\beta^{n+1} - \beta^{n-3}}{\beta - 1}.$$



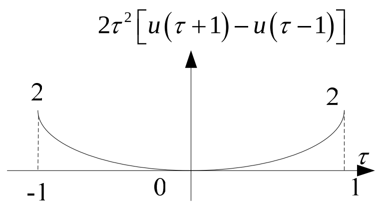
3.

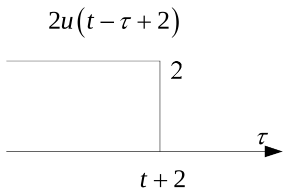
(1) $y(t) = 2t^2\left[u(t+1) - u(t-1)\right] * 2u(t+2)$.

For $t+2 < -1$, $t < -3$, $y(t) = 0$.

For $t+2 < 1$, $-3 < t < -1$, $y(t) = 2\int_{-1}^{t+2} 2\tau^2 d\tau = \dfrac{4}{3}\tau^3\Big|_{-1}^{t+2} = \dfrac{4}{3}\left[(t+2)^3 + 1\right]$.

For $t+2 \ge 1$, $-1 < t$, $y(t) = 2\int_{-1}^{1} 2\tau^2 d\tau = \dfrac{4}{3}\tau^3\Big|_{-1}^{1} = \dfrac{4}{3}[1+1] = \dfrac{8}{3}$





(2) $y(t) = e^{-\gamma t}u(t) * e^{\beta t}u(-t)$.

$y(t) = \int_0^{\infty} e^{-\gamma\tau} e^{\beta(t-\tau)} u(-t+\tau) d\tau$

For $t < 0$, $y(t) = \int_0^{\infty} e^{-\gamma\tau} e^{\beta(t-\tau)} d\tau = e^{\beta t}\int_0^{\infty} e^{-(\gamma+\beta)\tau} d\tau = e^{\beta t}/(\beta+\gamma)$.

For $t \ge 0$,

$$y(t) = \int_t^{\infty} e^{-\gamma\tau} e^{\beta(t-\tau)} d\tau = e^{\beta t}\int_t^{\infty} e^{-(\gamma+\beta)\tau} d\tau$$
$$= e^{\beta t} e^{-(\gamma+\beta)t}/(\beta+\gamma) = e^{-\gamma t}/(\beta+\gamma)$$

4.

- The system is memoryless if and only if $h(t) = c\delta(t)$ or $h[n] = c\delta[n]$.
- The system is causal if and only if $h(t) = 0$ for $t < 0$ or $h[n] = 0$ for $n < 0$.
- The system is stable if and only if $\int_{-\infty}^{\infty} |h(t)| dt < \infty$ or $\sum_{k=-\infty}^{\infty} |h[k]| < \infty$.

(1) $h(t) = \cos(\pi t)$

Has Memory. Not causal ($h(-1) = \cos(-\pi) = -1 \neq 0$). Not sstable.

$$\begin{aligned}\int_{-\infty}^{\infty} |\cos(\pi t)| dt &= 2\int_{0}^{\infty} |\cos(\pi t)| dt \\ &= 2 \lim_{N\to\infty} N \left(\int_{0}^{1/2} \cos(\pi t) dt + \int_{1/2}^{1} |\cos(\pi t)| dt \right) \\ &= 4 \lim_{N\to\infty} N \int_{0}^{1/2} \cos(\pi t) dt = 4 \lim_{N\to\infty} N \cdot 1 = \infty\end{aligned}$$

(2) $h(t) = e^{-2t} u(t-1)$

Has memory ($h(t) \neq c\delta(t)$). Causal. Stable.

(3) $h[n] = (1/2)^{|n|}$

Has memory ($h[n] \neq c\delta[n]$). Not causal ($h[-1] = (1/2)$). Stable.

(4) $h[n] = \sum_{p=-1}^{\infty} \delta[n - 2p]$

Has memory ($h[n] \neq c\delta[n]$).

Not causal ($h[-2] = \sum_{p=-1}^{\infty} \delta[-2 - 2p] = 1 \neq 0$).

Not stable. $$\begin{aligned}\sum_{k=-\infty}^{\infty} \left| \sum_{p=-1}^{\infty} \delta[k-2p] \right| &= \sum_{k=-\infty}^{\infty} \sum_{p=-1}^{\infty} \delta[k-2p] \\ &= \sum_{\substack{k=-1 \\ k \text{ is even.}}}^{\infty} 1 = \infty\end{aligned}$$

5.

(1) $h[n] = (-1)^n \{u[n+2] - u[n-3]\}$. The step response is

$$s[n] = u[n] * h[n] = \sum_{k=-\infty}^{n} h[k]$$

For $n < -2$, $s[n] = 0$.

For $-2 \le n \le 2$, $s[n] = \begin{cases} 1, & n = \pm 2, 0 \\ 0, & n = \pm 1 \end{cases}$.

For $n \ge 3$, $s[n] = 1$

(2) $h(t) = e^{-|t|}$. The step response is

$s(t) = u(t) * h(t) = \int_{-\infty}^{t} h(\tau) d\tau$

For $t < 0$, $\int_{-\infty}^{t} e^{\tau} d\tau = e^{t}$.

For $t \ge 0$, $\int_{-\infty}^{0} e^{\tau} d\tau + \int_{0}^{t} e^{-\tau} d\tau = 1 + 1 - e^{-t} = 2 - e^{-t}$