Homework No. 1 Solution

1. (20%)

(1)

$$\int_{-a}^{a} x(t)\,dt = \int_{-a}^{0} x(t)\,dt + \int_{0}^{a} x(t)\,dt$$
$$= \underbrace{\int_{a}^{0} x(-\lambda)\,d(-\lambda)}_{t=-\lambda} + \int_{0}^{a} x(t)\,dt = \int_{0}^{a} x(-\lambda)\,d\lambda + \int_{0}^{a} x(t)\,dt$$
$$= \underbrace{\int_{0}^{a} x(\lambda)\,d\lambda}_{even,\ x(-\lambda)=x(\lambda)} + \int_{0}^{a} x(t)\,dt = 2\int_{0}^{a} x(t)\,dt$$

$$\sum_{n=-k}^{k} x[n] = \sum_{n=-k}^{-1} x[n] + x[0] + \sum_{n=1}^{k} x[n]$$
$$= \underbrace{\sum_{m=k}^{1} x[-m]}_{n=-m} + x[0] + \sum_{n=1}^{k} x[n]$$
$$= \underbrace{\sum_{m=1}^{k} x[m]}_{even,\ x[-m]=x[m]} + x[0] + \sum_{n=1}^{k} x[n] = x[0] + 2\sum_{n=1}^{k} x[n]$$

(2) Since $x(t)$ and $x[n]$ are odd, that is, $x(t) = -x(-t)$ and $x[n] = -x[-n]$, we have

$$x(0) = -x(-0) \text{ and } x[0] = -x[-0]$$

Hence,

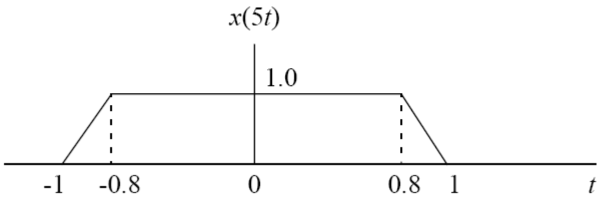
$$x(0) = -x(-0) = -x(0) \Rightarrow x(0) = 0$$
$$x[0] = -x[-0] = -x[0] \Rightarrow x[0] = 0$$

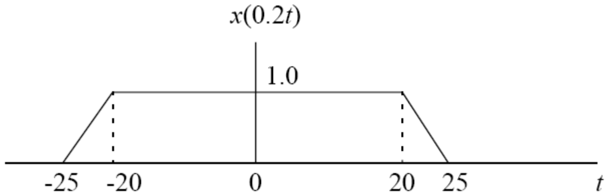
$$\int_{-a}^{a} x(t)\,dt = \int_{-a}^{0} x(t)\,dt + \int_{0}^{a} x(t)\,dt$$
$$= \underbrace{\int_{a}^{0} x(-\lambda)\,d(-\lambda)}_{t=-\lambda} + \int_{0}^{a} x(t)\,dt = \underbrace{\int_{0}^{a} -x(\lambda)\,d\lambda}_{odd,\ -x(-\lambda)=x(\lambda)} + \int_{0}^{a} x(t)\,dt = 0$$

$$\sum_{n=-k}^{k} x[n] = \sum_{n=-k}^{-1} x[n] + x[0] + \sum_{n=1}^{k} x[n]$$
$$= \underbrace{\sum_{m=k}^{1} x[-m]}_{n=-m} + x[0] + \sum_{n=1}^{k} x[n]$$
$$= \underbrace{\sum_{m=1}^{k} -x[m]}_{odd,\ -x[-m]=x[m]} + x[0] + \sum_{n=1}^{k} x[n] = x[0] = 0$$

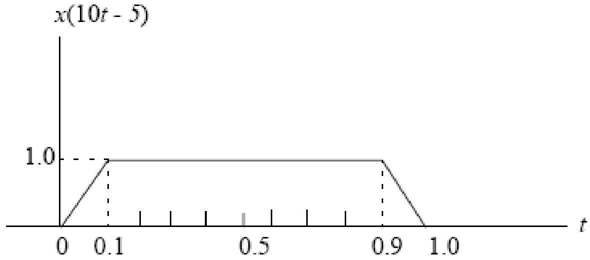
2. (10%)

(1)





(2)



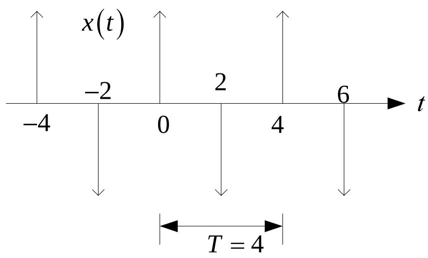
3. (30%)

(1) $x[n] = \cos\left(\frac{8}{15}\pi n\right) \Rightarrow N = \frac{2\pi m}{8\pi/15} = \frac{15m}{4} = 15$ samples $(m = 4)$, periodic

(2) $x(t) = \cos(2t) + \sin(3t)$

$$\left.\begin{aligned} 2t &= 2\pi\frac{1}{T_1}t \Rightarrow T_1 = \pi \\ 3t &= 2\pi\frac{1}{T_2}t \Rightarrow T_2 = \frac{2\pi}{3} \end{aligned}\right\} \Rightarrow T_0 = 2\pi \text{ , periodic}$$

(3) $x(t) = \sum_{k=-\infty}^{\infty}(-1)^k \delta(t - 2k)$



Periodic,

(4)

$$\begin{aligned} x(t) &= v(t) + v(-t) \\ &= \cos(t)u(t) + \cos(-t)u(-t) \\ &= \cos(t)\left[u(t) + u(-t)\right] \end{aligned}$$

From the definition of unit function in the textbook, the point $t = 0$ is undefined, so $\cos(t)\left[u(t) + u(-t)\right]$ is nonperiodic. But some books usually give $\cos(t)\left[u(t) + u(-t)\right] = \cos(t)$ and its period is

$$t = 2\pi \frac{1}{T} t \Rightarrow T = 2\pi \text{ sec}.$$

(5) Nonperiodic

$$\begin{aligned} x(t) &= v(t) + v(-t) \\ &= \sin(t)u(t) + \sin(-t)u(-t) \\ &= \sin(t)\left[u(t) - u(-t)\right] \end{aligned}$$

(6) Periodic

$$x[n] = \cos\left(\frac{1}{5}\pi n\right)\sin\left(\frac{1}{3}\pi n\right) = \frac{1}{2}\left\{\sin\left(\frac{8}{15}\pi n\right) + \sin\left(\frac{2}{15}\pi n\right)\right\}$$

$$\left.\begin{aligned} &\frac{8}{15}\pi N = 2\pi m \Rightarrow N = \frac{15}{4} m = 15, 30, \ldots \\ &\frac{2}{15}\pi N = 2\pi l \Rightarrow N = 15 l = 15, 30, \ldots \end{aligned}\right\} \Rightarrow N = 15 \text{ samples}$$

4.

(1) (10%)

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t) dt$$

Allowing the limit to be taken in a manner such that *T* is an integer multiple of the fundamental period, $T = kT_0$, the total normalized energy content of $x(t)$ over an interval of length *T* is *k* times the normalized energy content over one period. Then

$$P = \lim_{k \to \infty} \left[\frac{1}{kT_0} \int_{-kT_0/2}^{kT_0/2} x^2(t) dt \right] = \lim_{k \to \infty} \left[\frac{1}{kT_0} k \int_{-T_0/2}^{T_0/2} x^2(t) dt \right] = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x^2(t) dt$$

(2) (20%)

(a) $x(t)$ is an energy signal.

$$E = \int_{-\infty}^{\infty} x^2(t)\, dt = \int_0^{\infty} e^{-2at} dt = -\frac{1}{2a} e^{-2at} \Big|_0^{\infty} = \frac{1}{2a} < \infty$$

(b) $x(t) = A\cos(\omega_0 t + \theta)$, $T_0 = 2\pi/\omega_0$

$$\begin{aligned} P &= \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x^2(t) dt = \frac{\omega_0}{2\pi} \int_{-\pi/\omega_0}^{\pi/\omega_0} A^2 \cos^2(\omega_0 t + \theta) dt \\ &= \frac{\omega_0 A^2}{2\pi} \int_{-\pi/\omega_0}^{\pi/\omega_0} \frac{1}{2}\left[1 + \cos(2\omega_0 t + 2\theta)\right] dt = \frac{A^2}{2} < \infty \end{aligned}$$

$x(t)$ is a power signal.

(c) $x(t) = tu(t)$

$$E = \lim_{T\to\infty}\int_0^{T/2} t^2(t)dt = \lim_{T\to\infty}\frac{(T/2)^3}{3} = \infty$$

$$P = \lim_{T\to\infty}\frac{1}{T}\int_0^{T/2} t^2(t)dt = \lim_{T\to\infty}\left[\frac{1}{T}\times\frac{(T/2)^3}{3}\right] = \lim_{T\to\infty}\frac{T^2}{24} = \infty$$

$x(t)$ is neither an energy signal nor a power signal.

(d) $x[n] = (-0.5)^n u[n]$ is an energy signal.

$$E = \sum_{n=0}^{\infty}(0.25)^n = \frac{1}{1-0.25} = \frac{4}{3} < \infty$$

(e) $x[n] = u[n]$ is a power signal.

$$P = \lim_{N\to\infty}\frac{1}{2N}\sum_{n=0}^{N} 1^2 = \lim_{N\to\infty}\frac{1}{2N}(N+1) = \frac{1}{2} < \infty$$

5. (10%)

(1) $\int_{-\infty}^{\infty}\left(t^2+\cos(\pi t)\right)\delta(t-1)dt = \left(t^2+\cos(\pi t)\right)\Big|_{t=1} = 1+\cos(\pi) = 1-1 = 0$

(2) $\int_{-\infty}^{\infty} e^{-t}\delta(2t-2)dt = \int_{-\infty}^{\infty} e^{-t}\delta\left(2(t-1)\right)dt = \int_{-\infty}^{\infty} e^{-t}\frac{1}{2}\delta(t-1)dt = \frac{1}{2}e^{-t}\Big|_{t=1} = \frac{1}{2e}$