Homework No. 1

Due 10:10 am, March 14, 2006

1. Show that

   (1) If $x(t)$ and $x[n]$ are even, then

$$\int_{-a}^{a} x(t)\,dt = 2\int_{0}^{a} x(t)\,dt; \quad \sum_{n=-k}^{k} x[n] = x[0] + 2\sum_{n=1}^{k} x[n]$$

   (2) If $x(t)$ and $x[n]$ are odd, then

$$x(0) = 0 \text{ and } x[n] = 0; \quad \int_{-a}^{a} x(t)\,dt = 0 \quad \text{and} \quad \sum_{n=-k}^{k} x[n] = 0$$

2.

   (1) The trapezoidal pulse $x(t)$ of Fig. 1 is time scaled, producing the equation $y(t) = x(at)$. Sketch $y(t)$ for $a = 5$ and 0.2.

   (2) Sketch the trapezoidal pulse $y(t)$ related to that of Fig. 1 as follows

$$y(t) = x(10t - 5)$$

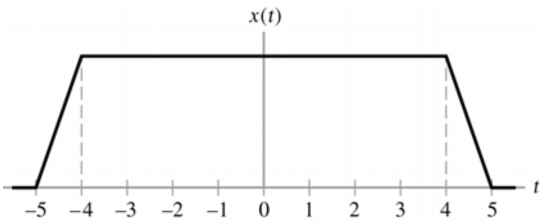

Figure 1

3. Determine whether the following signals are periodic, and for those which are, find the fundamental period:

   (1) $x[n] = \cos(\frac{8}{15}\pi n)$; (2) $x(t) = \cos(2t) + \sin(3t)$;

   (3) $x(t) = \sum_{k=-\infty}^{\infty} (-1)^k \delta(t - 2k)$; (4) $x(t) = v(t) + v(-t)$, where $v(t) = \cos(t)u(t)$;

   (5) $x(t) = v(t) + v(-t)$, where $v(t) = \sin(t)u(t)$; (6) $x[n] = \cos(\frac{1}{5}\pi n)\sin(\frac{1}{3}\pi n)$.

4.

   (1) Show that if $x(t)$ is periodic with period T_0, then the time-averaged power P of $x(t)$ is the same as the average power of $x(t)$ over any interval of length T_0. That is, $P = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x^2(t)\,dt$.

   (2) Determine whether the following signals are energy signals, power signals, or neither.

   (a) $x(t) = e^{-at}u(t),\ a > 0$; (b) $x(t) = A\cos(\omega_0 t + \theta)$;

(c) $x(t) = tu(t)$; (d) $x[n] = (-0.5)^n u[n]$; (e) $x[n] = u[n]$.

5. Evaluate following integrals:

(1) $\int_{-\infty}^{\infty} \left(t^2 + \cos(\pi t)\right)\delta(t-1)\,dt$

(2) $\int_{-\infty}^{\infty} e^{-t}\delta(2t-2)\,dt$