Homework No. 7 Solution
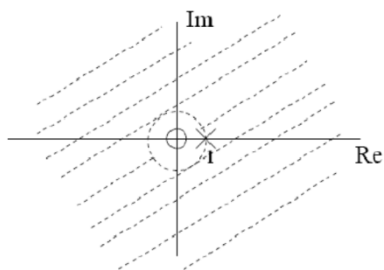
1.

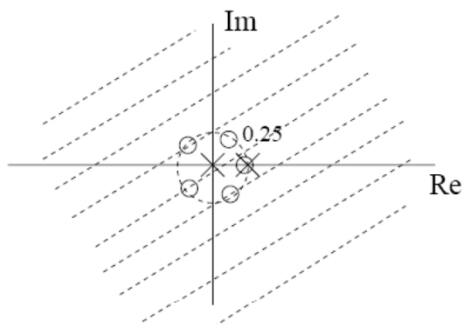(a)

$x[n] = u[n]$

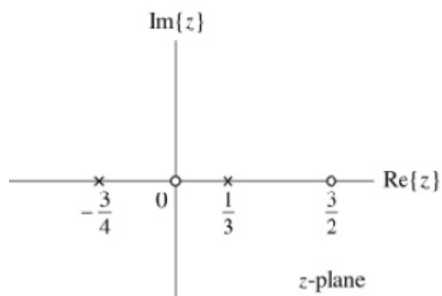$$X(z) = \sum_{n=0}^{\infty} z^{-n}$$

$$= \frac{1}{1 - z^{-1}}, \quad |z| > 1$$

(b)

$$X(z) = \sum_{n=0}^{4} \left(\frac{1}{4}z^{-1}\right)^{n}$$

$$= \frac{1 - \left(\frac{1}{4}z^{-1}\right)^{5}}{1 - \frac{1}{4}z^{-1}}$$

$$= \frac{\left[z^{5} - \left(\frac{1}{4}\right)^{5}\right]}{z^{4}\left(z - \frac{1}{4}\right)}, \quad \text{all } z$$

2.

(1) $|z| > \frac{3}{4}$

$x[n]$ is right-sided.

(2) $\frac{1}{3} < |z| < \frac{3}{4}$

$x[n]$ is two-sided.

(3) $|z| < \frac{1}{3}$

$x[n]$ is left-sided.

3.

(a)

$$X(z) = \frac{1 + \frac{7}{6}z^{-1}}{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 + \frac{1}{3}z^{-1}\right)}, \quad |z| > \frac{1}{2}$$

$$X(z) = \frac{2}{1 - \frac{1}{2}z^{-1}} + \frac{-1}{1 + \frac{1}{3}z^{-1}}$$

$$x[n] = \left[2(\frac{1}{2})^n - (-\frac{1}{3})^n\right]u[n]$$

(b)

$$X(z) = \frac{z^2 - 3z}{z^2 + \frac{3}{2}z - 1}, \quad \frac{1}{2} < |z| < 2$$

$$X(z) = \frac{-1}{1 - \frac{1}{2}z^{-1}} + \frac{2}{1 + 2z^{-1}}$$

$$x[n] = -\left(\frac{1}{2}\right)^n u[n] - 2(-2)^n u[-n-1]$$

4.

(a)

$$X(z) = \cos(z^{-3}), \quad |z| > 0$$

$$\cos(\alpha) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!}(\alpha)^{2k}$$

$$X(z) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!}(z^{-3})^{2k}$$

$$= \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!}z^{-6k}$$

$$x[n] = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!}\delta[n - 6k]$$

(b)

$$X(z) = \ln(1 + z^{-1}), \quad |z| > 0$$

$$\ln(1 + \alpha) = \sum_{k=0}^{\infty} \frac{(-1)^{k-1}}{k} (\alpha)^k$$

$$X(z) = \sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{k} (z^{-1})^k$$

$$x[n] = \sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{k} \delta[n - k]$$

5.

The corresponding inverse systems can be represented by

$$H_{I1}(z) = \frac{1 - \frac{1}{3}z^{-1}}{1 - 2z^{-1}}, \quad |z| > 2$$

and

$$H_{I2}(z) = \frac{1 - \frac{1}{3}z^{-1}}{1 - 2z^{-1}}, \quad |z| < 2.$$

Neither of $H_{I1}(z)$ and $H_{I2}(z)$ is both causal and stable, so there does not exist a both causal and stable inverse system for $H(z)$.