1.

(a) $x(t) = -e^{-at}u(t)$

$$X(s) = \frac{1}{s+a}, \text{Re}\{s\} > -a$$

(b) $x(t) = e^{-2t}u(t) + e^{-t}\cos(3t)u(t)$

$$X(s) = \frac{1}{s+2} + \frac{s+1}{(s+1)^2 + 3^2}, \text{Re}\{s\} > -1$$

2. $$x(t) = e^{-|t|} \overset{l}{\longleftrightarrow} X(s) = \frac{-2}{s^2-1}, -1 < \text{Re}\{s\} < 1$$

$$\therefore Y(s) = H(s)X(s) = \frac{s+1}{(s+1)^2+1}\frac{-2}{(s+1)(s-1)}$$

$$\therefore Y(s) = \frac{-2}{\left[(s+1)^2+1\right](s-1)} = \left\{\frac{As+B}{\left[(s+1)^2+1\right]} + \frac{c}{(s-1)}\right\}$$

$$\Rightarrow A = \frac{2}{5}, B = \frac{6}{5}, c = -\frac{2}{5}$$

$$y(t) = \frac{2}{5}e^{-t}\cos t + \frac{4}{5}e^{-t}\sin t - \frac{2}{5}e^{t}$$

3. $$H(s) = \frac{s^2+2s+2}{s^2-1} = 1 + \frac{2s+3}{s^2-1} = 1 + \left(\frac{\frac{5}{2}}{s+1} + \frac{\frac{-1}{2}}{s-1}\right)$$

(I) (a)Causal System $$h(t) = \delta(t) + \frac{5}{2}e^{-t}u(t) - \frac{1}{2}e^{t}u(t)$$

(b)Stable system $$h(t) = \delta(t) + \frac{5}{2}e^{-t}u(t) + \frac{1}{2}e^{t}u(-t)$$

(II) $$X(s) = \frac{1}{s+2}, Y(s) = \frac{-2}{s+1} + \frac{2}{s+3})$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{-4(s+2)}{(s+1)(s+3)} = \frac{-2}{s+1} + \frac{-2}{s+3}, \text{Re}\{s\} > -1$$

$$h(t) = -2e^{-t}u(t) + -2e^{-3t}u(t)$$

4.

(1) $x(t-1) \overset{\mathcal{L}}{\longleftrightarrow} e^{-1s}X(s) = e^{-1s}\frac{2s}{s^2+2}$.

(2) $e^{-3t}x(t) \overset{\mathcal{L}}{\longleftrightarrow} X(s+3) = \frac{2(s+3)}{(s+3)^2+2}$

(3) $x(t) * \frac{d}{dt}x(t)$

$$b(t) = \frac{d}{dt}x(t) \overset{\mathcal{L}}{\longleftrightarrow} B(s) = sX(s)$$

$$y(t) = x(t) * b(t) \overset{\mathcal{L}}{\longleftrightarrow} Y(s) = X(s)B(s) = sX^2(s) = s\left(\frac{2s}{s^2+2}\right)^2.$$

(4) $\int_0^t x(3\tau)\,d\tau \overset{\mathcal{L}}{\longleftrightarrow} Y(s) = \frac{X(s/3)}{3s} = \frac{2}{s^2+18}$

5.

$$\int_{-\infty}^{\infty} e^{-10|t|}e^{-(\sigma+jw)t}dt = \int_{-\infty}^{0} e^{10t}e^{-(\sigma+jw)t}dt + \int_{0}^{\infty} e^{-10t}e^{-(\sigma+jw)t}dt$$

$$= \int_{-\infty}^{0} e^{-(-10+\sigma)t}e^{-(jw)t}dt + \int_{0}^{\infty} e^{-(10+\sigma)t}e^{-(jw)t}dt$$

The first integral converges for $-10+\sigma < 0 \Rightarrow \sigma < 10$. The second integral converges if $10+\sigma > 0 \Rightarrow \sigma > -10$. Therefore, the given integral converges when $|\sigma| < 10$.