Homework No. 5 Solution

1. Let $x[n]$ be a periodic signal with period N and Fourier coefficients a_k.

   (1) Express the Fourier coefficients b_k of $\left|x[n]\right|^2$ in terms of a_k. (10%)

   Since $x[n] \overset{F.S.}{\longleftrightarrow} a_k$ and $x[n] \overset{F.S.}{\longleftrightarrow} a_{-k}^*$. By using the convolution property, we have: $x[n]x^*[n] = \left|x[n]\right|^2 \overset{F.S.}{\longleftrightarrow} b_k = \sum_{l=<N>} a_l a_{l+k}^*$.

   (2) If the coefficients a_k are real, is it guaranteed that the coefficients b_k are also real? (10%)

   From (1), it is clear that the answer is yes.

2. You are given $x[n] = n(1/2)^{|n|} \overset{DTFT}{\longleftrightarrow} X(\Omega)$. Without evaluating $X(\Omega)$, find $y[n]$ if

   (1) $Y(\Omega) = \mathrm{Re}\{X(\Omega)\}$ (5%)

   $\Rightarrow$ Since $x[n]$ is real and odd, $X(\Omega)$ is pure imaginary, thus $y[n] = 0$.

   (2) $Y(\Omega) = dX(\Omega)/d\Omega$ (5%)

   $\Rightarrow y[n] = -jnx[n] = -jn^2(1/2)^{|n|}$.

   (3) $Y(\Omega) = X(\Omega) + X(-\Omega)$ (5%)

   $\Rightarrow y[n] = x[n] + x[-n] = n(1/2)^{|n|} - n(1/2)^{|n|} = 0$

   (4) $Y(\Omega) = e^{-4j\Omega}X(\Omega)$ (5%)

   $\Rightarrow y[n] = x[n-4] = (n-4)(1/2)^{|n-4|}$

3. Let $x[n]$ and $h[n]$ be the signals with the following Fourier transforms:

$$X\left(e^{j\Omega}\right) = 3e^{-j\Omega} + 1 - e^{j\Omega} + 2e^{j3\Omega}$$
$$H\left(e^{j\Omega}\right) = 2e^{-j2\Omega} - e^{-j\Omega} + e^{j4\Omega}$$

Determine $y[n] = x[n]*h[n]$. (15%)

$$\begin{aligned} y[n] &= x[n]*h[n] \\ &= \left(3\delta[n-1] + \delta[n] - \delta[n+1] + 2\delta[n+3]\right)*\left(2\delta[n-2] - \delta[n-1] + \delta[n+4]\right) \\ &= 6\delta[n-3] - \delta[n-2] - 3\delta[n-1] + \delta[n] + 4\delta[n+1] - 2\delta[n+2] + 3\delta[n+3] \\ &\quad + \delta[n+4] - \delta[n+5] + 2\delta[n+7] \end{aligned}$$

4. Consider the finite-length sequence $x[n] = 2\delta[n] + \delta[n-1] + \delta[n-3]$.

   (1) Compute the five-point DFT $X[k]$. (10%)

   $\Rightarrow X[k] = 2 + e^{-j\frac{2\pi}{5}k} + e^{-j3\frac{2\pi}{5}k}$.

   (2) If $Y[k] = X^2[k]$, determine the sequence $y[n]$ with five-point inverse DFT for $n = 0 \sim 4$. (10%)

   $$Y[k] = X^2[k] = 4 + 4e^{-j\frac{2\pi}{5}k} + e^{-j2\frac{2\pi}{5}k} + 4e^{-j3\frac{2\pi}{5}k} + 2e^{-j4\frac{2\pi}{5}k} + e^{-j6\frac{2\pi}{5}k}$$
   $$= 4 + 5e^{-j\frac{2\pi}{5}k} + e^{-j2\frac{2\pi}{5}k} + 4e^{-j3\frac{2\pi}{5}k} + 2e^{-j4\frac{2\pi}{5}k}$$

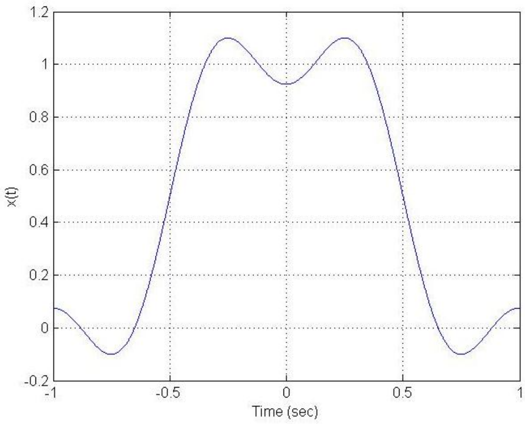
   $\therefore y[n] = 4\delta[n] + 5\delta[n-1] + \delta[n-2] + 4\delta[n-3] + 2\delta[n-4]$

   (3) If N-point DFTs are used here, how should we choose N such that $y[n] = x[n] * x[n]$, for $0 \le n \le N-1$. (5%)

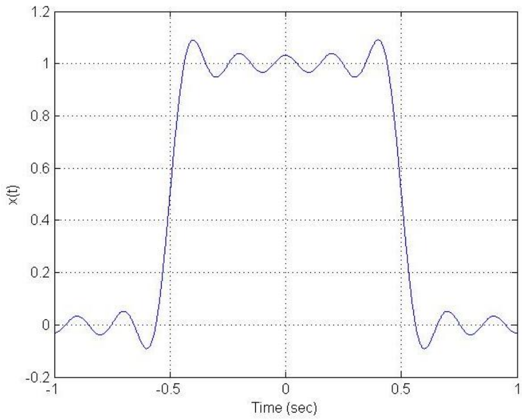
   $\Rightarrow$ N ≧ 4+4-1=7.

5. The plot of $x_N(t)$ for N = 3, 9, 21, and 45 are shown below, respectively. (20%)

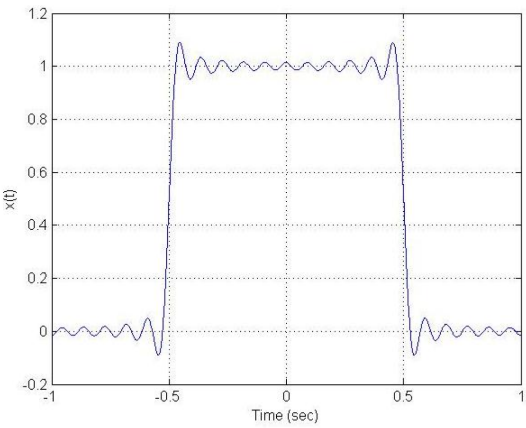
N = 3



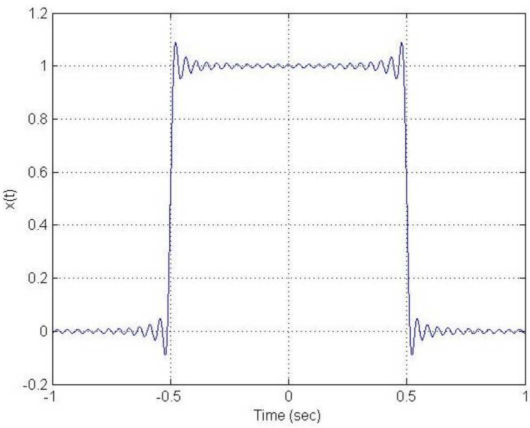
N = 9



N = 21



N = 45



Note that as N increases, the *ripples* in the approximating waveform $x_N(t)$ get narrower and narrower, but the *overshoot* of the waveform beyond the desired unity amplitude remains constant at about $\pm 0.18\%$, or about 9% of the height of the discontinuity .

MATLAB code:

```
clear all;
clc;
j = sqrt(-1);
t = -1:0.001:1;
N = input('Number of harmonics ');
a0 = 0.5; w0 = pi;
x = a0*ones(1,length(t));
for k = 1:2:N
    ak = sin((k*pi)/2)/(k*pi);
    x = x + 2*ak*exp(j*k*pi*t);
end
figure;
plot(t,x)
grid
ylabel('x(t)')
xlabel('Time (sec)')
```