Homework No. 5

Due 15:20, Dec. 5, 2012

1. Let $x[n]$ be a periodic signal with period N and Fourier coefficients a_k.
   (1) Express the Fourier coefficients b_k of $|x[n]|^2$ in terms of a_k. (10%)
   (2) If the coefficients a_k are real, is it guaranteed that the coefficients b_k are also real? (10%)

2. Given that $x[n]=n(1/2)^{|n|} \xleftrightarrow{DTFT} X(\Omega)$. Without evaluating $X(\Omega)$, find $y[n]$ if
   (1) $Y(\Omega)=\mathrm{Re}\{X(\Omega)\}$, (5%)
   (2) $Y(\Omega)=dX(\Omega)/d\Omega$, (5%)
   (3) $Y(\Omega)=X(\Omega)+X(-\Omega)$, (5%)
   (4) $Y(\Omega)=e^{-4j\Omega}X(\Omega)$. (5%)

3. Let $x[n]$ and $h[n]$ be the signals with the following Fourier transforms:

$$X\left(e^{j\Omega}\right)=3e^{-j\Omega}+1-e^{j\Omega}+2e^{j3\Omega},$$
$$H\left(e^{j\Omega}\right)=2e^{-j2\Omega}-e^{-j\Omega}+e^{j4\Omega}.$$

   Determine $y[n]=x[n]*h[n]$. (15%)

4. Consider the finite-length sequence $x[n]=2\delta[n]+\delta[n-1]+\delta[n-3]$.
   (1) Compute the five-point DFT $X[k]$. (10%)
   (2) If $Y[k]=X^2[k]$, determine the sequence $y[n]$ with five-point inverse DFT for $n = 0 \sim 4$. (10%)
   (3) If N-point DFTs are used here, how should we choose N such that $y[n]=x[n]*x[n]$, for $0\le n\le N-1$. (5%)

5. Consider a continuous-time signal $x(t)$ shown in Fig. 1. This signal is periodic with fundamental period $T_0=2$, and thus the fundamental frequency is $\omega_0=\pi$.

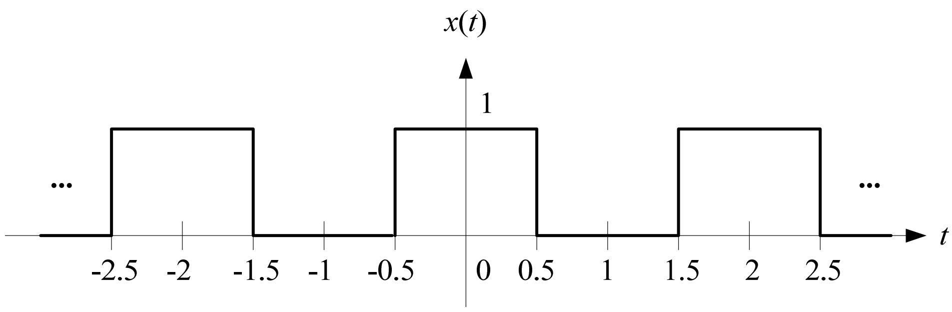

Fig. 1

The Fourier series representation of $x(t)$ is given by

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\pi t}, \text{ and } a_k = \begin{cases} 1/2, & \text{for } k = 0 \\ \dfrac{\sin(k\pi/2)}{k\pi}, & \text{for } k \neq 0 \end{cases}.$$

Since the Fourier series has an infinite number of terms, in general, it must be determined whether the series actually converges to $x(t)$ in a meaningful sense. Given a positive integer N, let $x_N(t)$ denote the finite series

$$x_N(t) = a_0 + \sum_{\substack{k=1 \\ k \text{ odd}}}^{N} 2a_k e^{jk\pi t}.$$

By Fourier's theorem, $x_N(t)$ should converge to $x(t)$ as $N \to \infty$. Plot $x_N(t)$ for $N = 3, 9, 21$, and 45 ranging t from −1 to 1 using **MATLAB**, and comment on the results. (20%)