

Homework No. 3
Due 16:30, Nov. 1, 2012

1. (20%) Identify the natural and forced responses for the systems described by the following difference equation with input and initial conditions as specified:

$$y[n] - \frac{1}{2}y[n-1] = 2x[n], \quad y[-1] = 3, \quad x[n] = \left(-\frac{1}{2}\right)^n u[n].$$

2. (20%) Determine the homogeneous and particular solutions for the system described by the following difference equation for the given inputs and initial conditions:

$$y[n] - \frac{1}{4}y[n-1] - \frac{1}{8}y[n-2] = x[n] + x[n-1], \quad y[-1] = 1, \quad y[-2] = 0.$$

- i. $x[n] = nu[n]$. (5%)
- ii. $x[n] = \left(\frac{1}{8}\right)^n u[n]$. (5%)
- iii. $x[n] = e^{j\frac{\pi}{4}n} u[n]$. (10%)
3. (20%) Determine the homogeneous and particular solutions for the systems described by the following differential equation for the given inputs and initial conditions:

$$\frac{d^2}{dt^2}y(t) + 4y(t) = 3\frac{d}{dt}x(t), \quad y(0^-) = -1, \quad \left.\frac{d}{dt}y(t)\right|_{t=0^-} = 1.$$

- i. $x(t) = t$. (5%)
- ii. $x(t) = e^{-t}$. (5%)
- iii. $x(t) = \sin(t) + \cos(t)$. (10%)
4. (20%) Consider a discrete-time LTI system described by the following difference equation for the given initial conditions:

$$y[n] - 5y[n-1] + 6y[n-2] = x[n] + x[n-1], \quad y[-1] = y[-2] = 0$$

- (a) Determine the complete solution $y[n]$ of the equation when $x[n] = u[n]$.
(10%)
- (b) What is the natural response $y^{(n)}[n]$ in this case? (5%)
- (c) What is the forced response $y^{(f)}[n]$ in this case? (5%)
5. (20%) Draw the direct form I and direct form II implementations for the following difference and differential equations:
- (a) $y[n] + \frac{1}{2}y[n-1] - y[n-3] = 3x[n-1] + 2x[n-2]$. (10%)
- (b) $\frac{d^3}{dt^3}y(t) + 2\frac{d}{dt}y(t) + 3y(t) = x(t) + 3\frac{d}{dt}x(t)$. (10%)