Homework No. 2 Solution

1.

(1) (30%) $x[n]=(-1)^n\left(u[n]-u[n-5]\right)$ and $h[n]=u[n+2]$.

$n+2<0,\ n<-2,\ w_n[k]=0,\ y[n]=0$

$0\le n+2\le 4,\ -2\le n\le 2,\quad w_n[k]=(-1)^k,\ 0\le k\le n+2$

$$y[n]=\sum_{k=0}^{n+2}(-1)^k=\begin{cases}1, & n \text{ is odd}\\ 0, & n \text{ is even}\end{cases}$$

$4<n+2,\ 2<n,\quad w_n[k]=(-1)^k,\ 0\le k\le 4$

$$y[n]=\sum_{k=0}^{4}(-1)^k=1$$

(2) (30%) $x[n]=u[n]-u[-n]$ and $h[n]=\begin{cases}\left(\frac{1}{2}\right)^n, & n\ge 0\\ 4^n, & n<0\end{cases}$.

$$h[n]=\begin{cases}\left(\frac{1}{2}\right)^n, & n\ge 0\\ 4^n, & n<0\end{cases}=\left(\frac{1}{2}\right)^n u[n]+4^n u[-n-1]$$

$$y[n]=x[n]*h[n]=u[n]*h[n]-u[-n]*h[n]$$

$$u[n]*h[n]=\sum_{k=-\infty}^{n}h[k]$$

$n\ge 0$,

$$\sum_{k=-\infty}^{n}h[k]=\sum_{k=-\infty}^{-1}4^k+\sum_{k=0}^{n}\left(\frac{1}{2}\right)^k$$

$$=\left(4^{-1}+4^{-2}+\cdots\right)+\left[1+\frac{1}{2}+\cdots+\left(\frac{1}{2}\right)^n\right]$$

$$=\frac{1}{3}+2\left[1-\left(\frac{1}{2}\right)^{n+1}\right]=\frac{7}{3}-\left(\frac{1}{2}\right)^n$$

$n<0$,

$$\sum_{k=-\infty}^{n} h[k] = \sum_{k=-\infty}^{n} 4^k = 4^n + 4^{n-1} + \cdots$$
$$= 4^n\left(1 + 4^{-1} + \cdots\right) = \frac{4}{3}4^n$$

$$u[-n] * h[n] = \sum_{k=n}^{\infty} h[k]$$

$n \ge 0$,

$$\sum_{k=n}^{\infty} h[k] = \sum_{k=n}^{\infty}\left(\frac{1}{2}\right)^k$$
$$= \left(\frac{1}{2}\right)^n + \left(\frac{1}{2}\right)^{n+1} + \cdots = \left(\frac{1}{2}\right)^n\left(1 + \frac{1}{2} + \cdots\right) = 2\left(\frac{1}{2}\right)^n$$

$n < 0$,

$$\sum_{k=n}^{\infty} h[k] = \sum_{k=n}^{-1} 4^k + \sum_{k=0}^{\infty}\left(\frac{1}{2}\right)^k$$
$$= 4^{-1} + 4^{-2} + \cdots + 4^n + \left(1 + \frac{1}{2} + \cdots\right)$$
$$= 4^{-1}\left(1 + 4^{-1} + \cdots + 4^{n+1}\right) + 2$$
$$= 4^{-1} \times \frac{4}{3} \times \left(1 - 4^n\right) + 2 = \frac{1}{3}\left(1 - 4^n\right) + 2 = \frac{7}{3} - \frac{4^n}{3}$$

$$y[n] = \left[\frac{7}{3} - \left(\frac{1}{2}\right)^n\right]u[n] + \frac{4}{3}4^n u[-n-1] - \left\{2\left(\frac{1}{2}\right)^n u[n] + \left(\frac{7}{3} - \frac{4^n}{3}\right)u[-n-1]\right\}$$
$$= \left[\frac{7}{3} - \left(\frac{1}{2}\right)^n - 2\left(\frac{1}{2}\right)^n\right]u[n] + \left\{\frac{4}{3}4^n - \left(\frac{7}{3} - \frac{4^n}{3}\right)\right\}u[-n-1]$$
$$= \left[\frac{7}{3} - 3\left(\frac{1}{2}\right)^n\right]u[n] + \left(\frac{5}{3}4^n - \frac{7}{3}\right)u[-n-1]$$

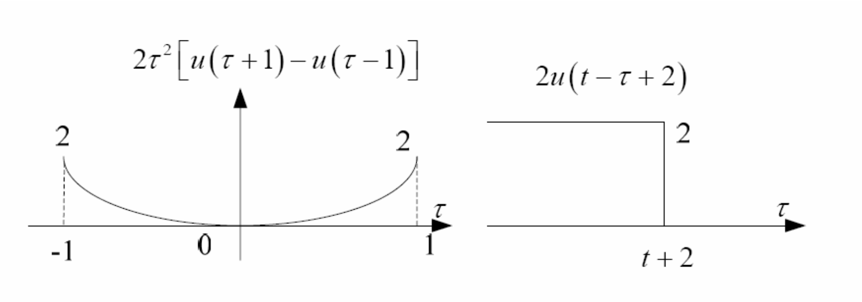
2. (20%)

$y(t) = 2t^2[u(t+1) - u(t-1)] * 2u(t+2).$

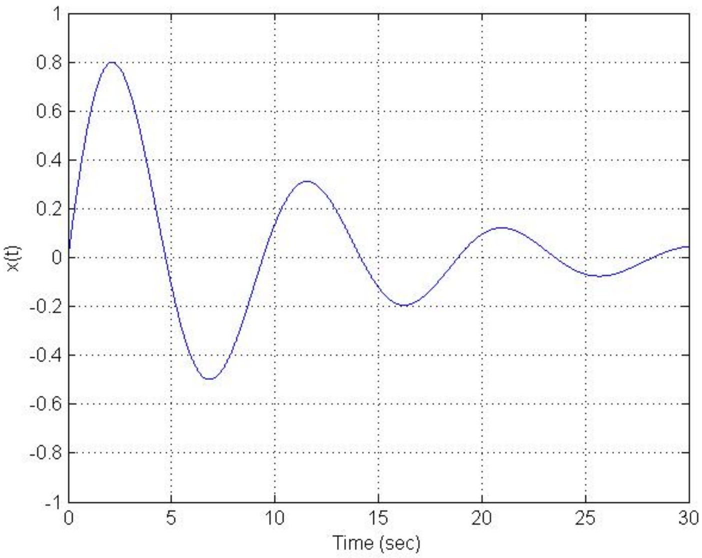
For $t + 2 < -1,\ t < -3,\ y(t) = 0.$

For $t + 2 < 1,\ \ -3 < t < -1,\ y(t) = 2\int_{-1}^{t+2} 2\tau^2 d\tau = \frac{4}{3}\tau^3\Big|_{-1}^{t+2} = \frac{4}{3}[(t+2)^3 + 1].$

For $t + 2 \ge 1,\ \ -1 < t,\ y(t) = 2\int_{-1}^{1} 2\tau^2 d\tau = \frac{4}{3}\tau^3\Big|_{-1}^{1} = \frac{4}{3}[1+1] = \frac{8}{3}.$



3. The plot of $x(t)$ is shown below. (10%)

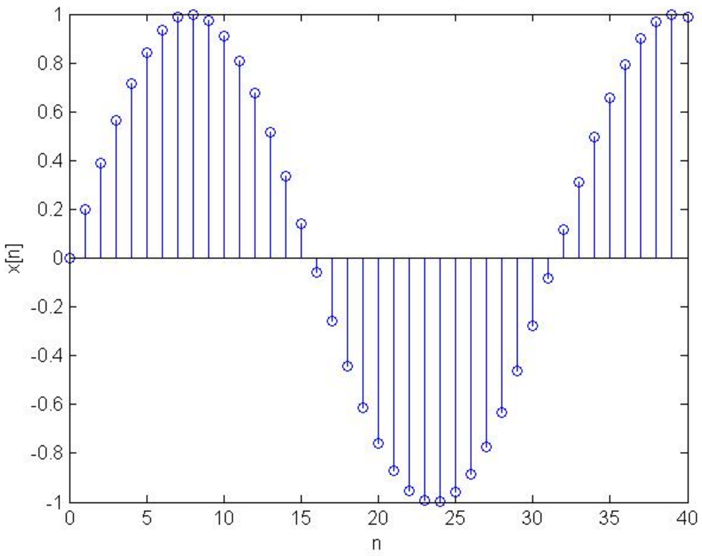


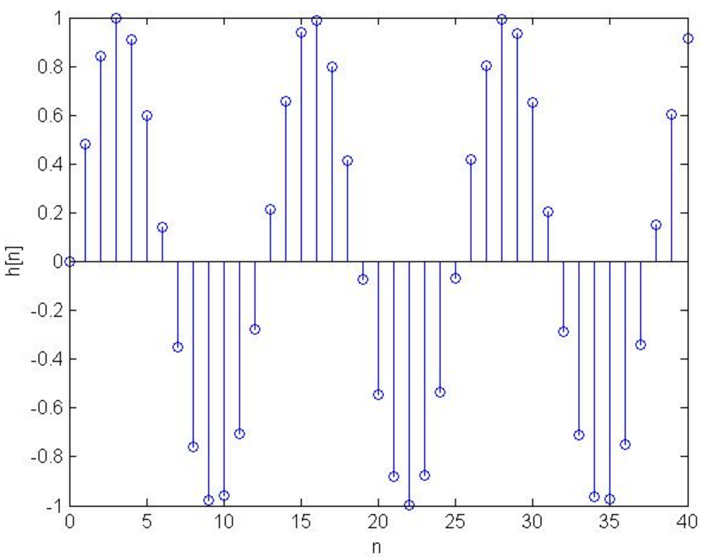
<u>MATLAB code</u>:

```
t = 0:0.01:30;
x = exp(-0.1*t).*sin(2/3*t);
plot(t,x)
grid
ylabel('x(t)')
xlabel('Time (sec)')
axis([0 30 -1 1]);
```

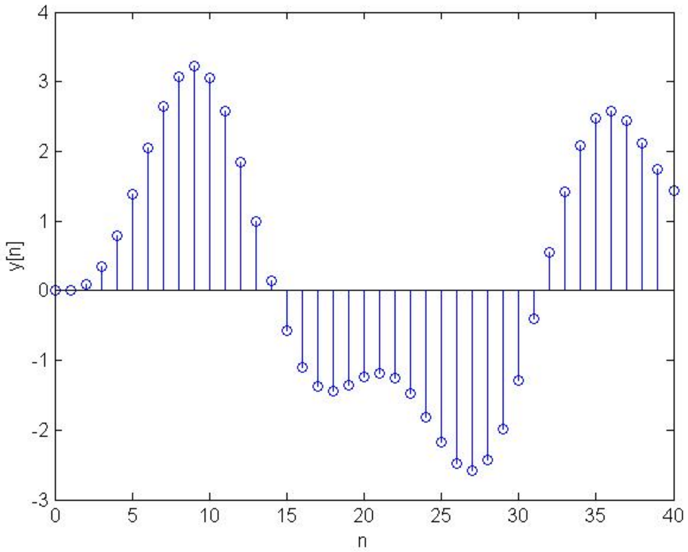
4.

(1) The plots of $x[n]$ and $h[n]$ are shown in the following. (20%)





(2) The plot of $y[n]$ is shown below. (20%)



<u>MATLAB Code</u>:

```
n = 0:40;
x = sin(0.2*n);
h = sin(0.5*n);
y = zeros(1,length(n));
for i = 1:41
    y(i) = 0;
    for j = 0:i-1
        y(i) = y(i) + x(j+1)*h(i-j);
    end
end
figure;
stem(n,x);
ylabel('x[n]')
xlabel('n')
figure;
stem(n,h);
ylabel('h[n]')
xlabel('n')
figure;
stem(n,y);
ylabel('y[n]')
xlabel('n')
```