

Homework No. 6 Solution

1.

$$(1) \quad x(t) = -e^{-at}u(t)$$

$$X(s) = \frac{1}{s+a}, \operatorname{Re}\{s\} > -a$$

$$(2) \quad x(t) = e^{-2t}u(t) + e^{-t}\cos(3t)u(t)$$

$$X(s) = \frac{1}{s+2} + \frac{s+1}{(s+1)^2 + 3^2}, \operatorname{Re}\{s\} > -1$$

$$2. \quad x(t) = e^{-|t|} \xrightarrow{\mathcal{L}} X(s) = \frac{-2}{s^2 - 1}, -1 < \operatorname{Re}\{s\} < 1$$

$$\therefore Y(s) = H(s)X(s) = \frac{s+1}{(s+1)^2 + 1} \frac{-2}{(s+1)(s-1)}$$

$$\therefore Y(s) = \frac{-2}{[(s+1)^2 + 1](s-1)} = \left\{ \frac{As+B}{[(s+1)^2 + 1]} + \frac{c}{(s-1)} \right\}$$

$$\Rightarrow A = \frac{2}{5}, B = \frac{6}{5}, c = -\frac{2}{5}$$

$$y(t) = \frac{2}{5}e^{-t}\cos t + \frac{4}{5}e^{-t}\sin t - \frac{2}{5}e^t$$

$$3. \quad H(s) = \frac{s^2 + 2s + 2}{s^2 - 1} = 1 + \frac{2s+3}{s^2 - 1} = 1 + \left(\frac{\frac{5}{2}}{s+1} + \frac{-1}{s-1} \right)$$

$$(1) \quad (a) \text{ Causal System} \quad h(t) = \delta(t) + \frac{5}{2}e^{-t}u(t) - \frac{1}{2}e^t u(t)$$

$$(b) \text{ Stable system} \quad h(t) = \delta(t) + \frac{5}{2}e^{-t}u(t) + \frac{1}{2}e^t u(-t)$$

$$(2) \quad X(s) = \frac{1}{s+2}, Y(s) = \frac{-2}{s+1} + \frac{2}{s+3}$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{-4(s+2)}{(s+1)(s+3)} = \frac{-2}{s+1} + \frac{-2}{s+3}, \operatorname{Re}\{s\} > -1$$

$$h(t) = -2e^{-t}u(t) + -2e^{-3t}u(t)$$

4.

$$(1) \quad x(t-1) \xleftrightarrow{\mathcal{L}} e^{-1s} X(s) = e^{-1s} \frac{2s}{s^2+2}.$$

$$(2) \quad e^{-3t} x(t) \xleftrightarrow{\mathcal{L}} X(s+3) = \frac{2(s+3)}{(s+3)^2+2}$$

$$(3) \quad x(t) * \frac{d}{dt} x(t)$$

$$b(t) = \frac{d}{dt} x(t) \xleftrightarrow{\mathcal{L}} B(s) = sX(s)$$

$$y(t) = x(t) * b(t) \xleftrightarrow{\mathcal{L}} Y(s) = X(s)B(s) = sX^2(s) = s \left(\frac{2s}{s^2+2} \right)^2.$$

$$(4) \quad \int_0^t x(3\tau) d\tau \xleftrightarrow{\mathcal{L}} Y(s) = \frac{X(s/3)}{3s} = \frac{2}{s^2+18}$$