Homework No. 1

Due 15:00, Oct. 5, 2011

1. Determine whether the following signals are periodic, and for those which are, find the fundamental period: (18%)

(1) $x[n]=\cos\left(\frac{8}{15}\pi n\right)$; (2) $x(t)=\cos(2t)+\sin(3t)$;

(3) $x(t)=\sum_{k=-\infty}^{\infty}(-1)^k\delta(t-2k)$; (4) $x(t)=v(t)+v(-t)$, where $v(t)=\cos(t)u(t)$;

(5) $x(t)=v(t)+v(-t)$, where $v(t)=\sin(t)u(t)$; (6) $x[n]=\cos\left(\frac{1}{5}\pi n\right)\sin\left(\frac{1}{3}\pi n\right)$.

2. A system consists of several subsystems connected as shown in Fig. 1. Find the operator *H* relating *x*(*t*) to *y*(*t*) for the following subsystem operators: (12%)

$$H_1: y_1(t)=x_1(t)x_1(t-1);$$
$$H_2: y_2(t)=|x_2(t)|;$$
$$H_3: y_3(t)=1+2x_3(t);$$
$$H_4: y_4(t)=\cos(x_4(t)).$$

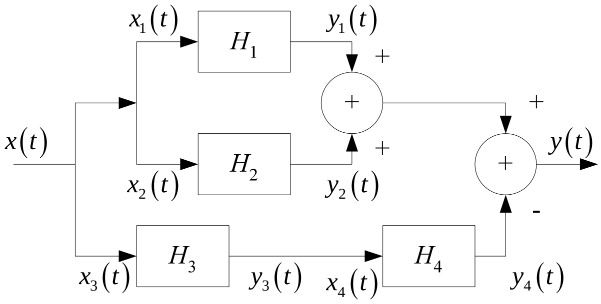

Figure 1

3. The system that follow have input *x*(*t*) or *x*[*n*] and output *y*(*t*) or *y*[*n*]. For each system, determine whether it is (i) memoryless, (ii) stable, (iii) causal, (iv) linear, and (v) time invariant. (25%)

(1) $y(t)=\cos(x(t))$; (2) $y[n]=2x[n]u[n]$; (3) $y[n]=\log_{10}(|x[n]|)$;

(4) $y(t)=\int_{-\infty}^{t/2}x(\tau)d\tau$; (5) $y[n]=\sum_{k=-\infty}^{n}x[k+2]$.

4. The output of a discrete-time system is related to its input $x[n]$ as follows:

$$y[n] = a \cdot x[n] - b \cdot x[n-1] + c \cdot x^2[n-2]$$

Let the operator S^k denote a system that shifts the input $x[n]$ by k time units to produce $x[n-k]$. Formulate the operator H for the system relating $y[n]$ to $x[n]$. Then develop a block diagram representation for H, using (a) cascade implementation and (b) parallel implementation. (10%)

5. Show that

(1) If $x(t)$ and $x[n]$ are even, then

$$\int_{-a}^{a} x(t)\,dt = 2\int_{0}^{a} x(t)\,dt; \quad \sum_{n=-k}^{k} x[n] = x[0] + 2\sum_{n=1}^{k} x[n] \qquad (10\%)$$

(2) If $x(t)$ and $x[n]$ are odd, then

$$x(0) = 0 \text{ and } x[n] = 0; \quad \int_{-a}^{a} x(t)\,dt = 0 \quad \text{and} \quad \sum_{n=-k}^{k} x[n] = 0 \qquad (10\%)$$

6 (15%)

(1) The trapezoidal pulse $x(t)$ of Fig. 2 is time scaled, producing the equation $y(t) = x(at)$. Sketch $y(t)$ for a = 10 and 0.5.

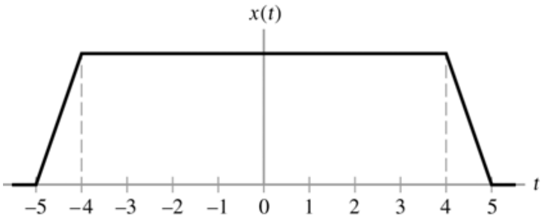

Figure 2

(2) Sketch the trapezoidal pulse $y(t)$ related to that of Fig. 2 as follows

$$y(t) = x(5(t-1))$$