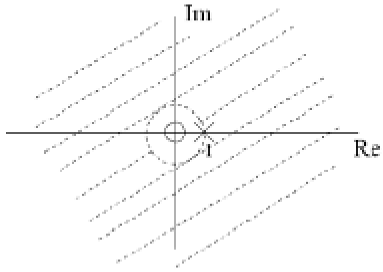
Homework No. 6 Solution

1.

(a)

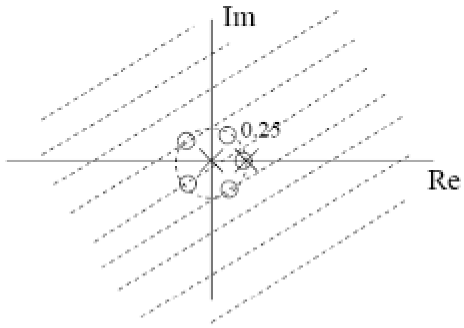
$x[n] = u[n]$

$$X(z) = \sum_{n=0}^{\infty} z^{-n} = \frac{1}{1-z^{-1}}, \quad |z| > 1$$

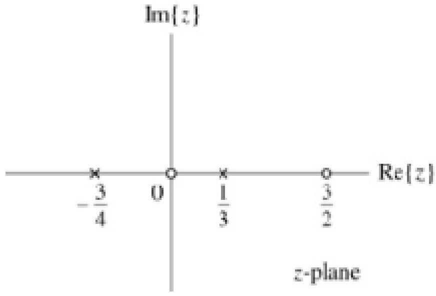


(b)

$$X(z) = \sum_{n=0}^{4} \left(\frac{1}{4}z^{-1}\right)^n = \frac{1-\left(\frac{1}{4}z^{-1}\right)^5}{1-\frac{1}{4}z^{-1}} = \frac{\left[z^5-\left(\frac{1}{4}\right)^5\right]}{z^4\left(z-\frac{1}{4}\right)}, \quad \text{all } z$$



2.



(1) $|z| > \frac{3}{4}$
$x[n]$ is right-sided.

(2) $\frac{1}{3} < |z| < \frac{3}{4}$
$x[n]$ is two-sided.

(3) $|z| < \frac{1}{3}$
$x[n]$ is left-sided.

3.

(a)

$$X(z) = \frac{1+\frac{7}{6}z^{-1}}{\left(1-\frac{1}{2}z^{-1}\right)\left(1+\frac{1}{3}z^{-1}\right)}, \quad |z| > \frac{1}{2}$$

$$X(z) = \frac{2}{1-\frac{1}{2}z^{-1}} + \frac{-1}{1+\frac{1}{3}z^{-1}}$$

$$x[n] = \left[2(\frac{1}{2})^n - (-\frac{1}{3})^n\right]u[n]$$

(b)

$$X(z) = \frac{z^2-3z}{z^2+\frac{3}{2}z-1}, \quad \frac{1}{2} < |z| < 2$$

$$X(z) = \frac{-1}{1-\frac{1}{2}z^{-1}} + \frac{2}{1+2z^{-1}}$$

$$x[n] = -\left(\frac{1}{2}\right)^n u[n] - 2(-2)^n u[-n-1]$$

4.

(a)

$$X(z) = \cos\left(z^{-3}\right), \quad |z| > 0$$

$$\cos(\alpha) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!}(\alpha)^{2k}$$

$$X(z) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!}(z^{-3})^{2k} = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!}z^{-6k}$$

$$x[n] = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!}\delta[n-6k]$$

(b)

$$X(z) = \ln(1 + z^{-1}), \quad |z| > 0$$

$$\ln(1+\alpha) = \sum_{k=0}^{\infty} \frac{(-1)^{k-1}}{k}(\alpha)^k$$

$$X(z) = \sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{k}(z^{-1})^k$$

$$x[n] = \sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{k}\delta[n-k]$$

5.

The corresponding inverse systems can be represented by

$$H_{I1}(z) = \frac{1 - \frac{1}{3}z^{-1}}{1 - 2z^{-1}}, \quad |z| > 2$$

and

$$H_{I2}(z) = \frac{1 - \frac{1}{3}z^{-1}}{1 - 2z^{-1}}, \quad |z| < 2.$$

Neither of $H_{I1}(z)$ and $H_{I2}(z)$ is both causal and stable, so there does not exist a both causal and stable inverse system for $H(z)$.