**Solution of Homework No. 5**

1.

Let us call the given impulse response h[n]. It is easily observed that the signal $h_1[n]=h[n+2]$ is real and even. Therefore (using properties of the Fourier transform) we know that the Fourier transform $H_1(jw)$ of h[n] is real and even. Therefore $H_1(jw)$ has zero phase. We also know that the Fourier transform $H(jw)=H_1(jw)e^{-2jw}$.

$$\measuredangle H(jw) = -2w, \qquad \text{group delay } \tau(w) = \frac{d}{dw}\measuredangle H(jw) = 2$$

2.

Note that $H(jw)=-2jw$.

(a) Here, $x(t)=e^{jt}$. Therefore, $y(t)=-2dx(t)/dt=-2je^{jt}$.
Or, by using Eigen function of LTI systems. Then $x(t)=e^{jt}$, $y(t)$ should be $H(jt)e^{jt}=-2je^{jt}$.

(b) $x(t)=\sin(w_0t)u(t)$. Then,
$dx(t)/dt=w_0\cos(w_0t)u(t)+\sin(w_0t)\delta(t)=w_0\cos(w_0t)u(t)$.
Therefore, $y(t)=-2dx(t)/dt=-2w_0\cos(w_0t)u(t)$.

(c) Here, $H(jw)=1/(2+jw)$. From this we obtain $x(t)=e^{-2t}u(t)$.
Therefore, $y(t)=-2dx(t)/dt=4e^{-2t}u(t)-2\delta(t)$.

3.

$$H(s)=\frac{s^2+2s+2}{s^2-1}=1+\frac{2s+3}{s^2-1}=1+\left(\frac{\frac{5}{2}}{s+1}+\frac{\frac{-1}{2}}{s-1}\right)$$

(a)Causal System $\quad h(t)=\delta(t)+\frac{5}{2}e^{-t}u(t)-\frac{1}{2}e^{t}u(t)$

(b)Stable system $\quad h(t)=\delta(t)+\frac{5}{2}e^{-t}u(t)+\frac{1}{2}e^{t}u(-t)$

4. Find the Laplace transform of following signals; indicate the ROC of each signal with figure also. (15%)

(a) $x(t)=-e^{-at}u(t)$

$$X(s)=\frac{1}{s+a}, \mathrm{Re}\{s\}>-a$$

(b) $x(t)=e^{-2t}u(t)+e^{-t}\cos(3t)u(t)$

$$X(s) = \frac{1}{s+2} + \frac{s+1}{(s+1)^2 + 3^2}, \mathrm{Re}\{s\} > -1$$

(c) $x(t) = \delta(t) - \frac{4}{3}e^{-t}u(t) + \frac{1}{3}e^{2t}u(t)$

$$X(s) = 1 - \frac{4}{3}\frac{1}{s+1} + \frac{1}{3}\frac{1}{s-2}, \mathrm{Re}\{s\} > 2$$