Homework No. 1 Solution

1. (20%)

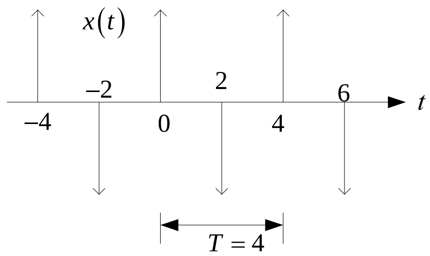
(1) Periodic and period=7.

$$\frac{6}{7}\pi N = 2\pi m \Rightarrow N = \frac{7}{6}m = 7 \text{ if } m = 6.$$

(2) $x(t) = \left[\cos\left(2t - \frac{\pi}{3}\right)\right]^2 = \dfrac{1+\cos\left(4t - \frac{2\pi}{3}\right)}{2}$

Periodic and period $= 2\pi/4 = \pi/2$

(3) $x(t) = \sum_{k=-\infty}^{\infty} (-1)^k \delta(t-2k)$



Periodic,

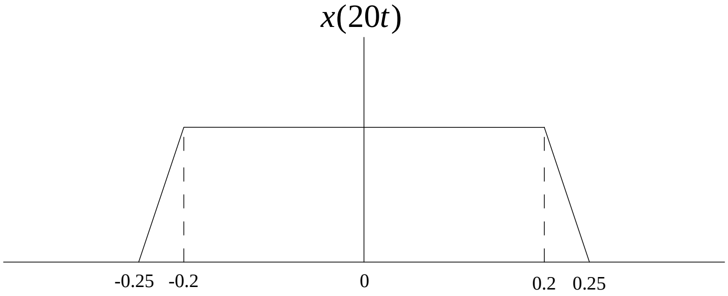
(4) Periodic

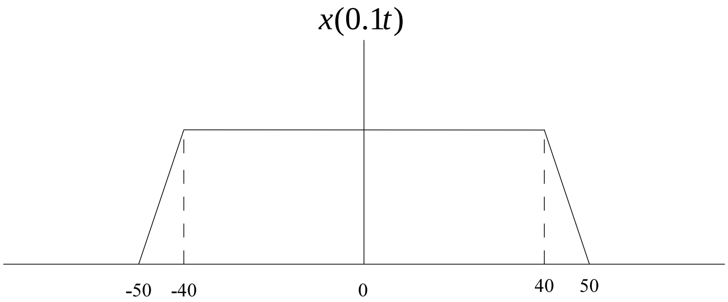
$$x[n] = 0.5\left[\cos(3\pi n/4) + \cos(\pi n/4)\right]$$

$$\left.\begin{aligned} &\frac{3}{4}\pi N = 2\pi m \Rightarrow N = \frac{8}{3}m = 8,16,\ldots \\ &\frac{1}{4}\pi N = 2\pi l \Rightarrow N = 8l = 8,16,\ldots \end{aligned}\right\} \Rightarrow N = 8 \text{ samples}$$

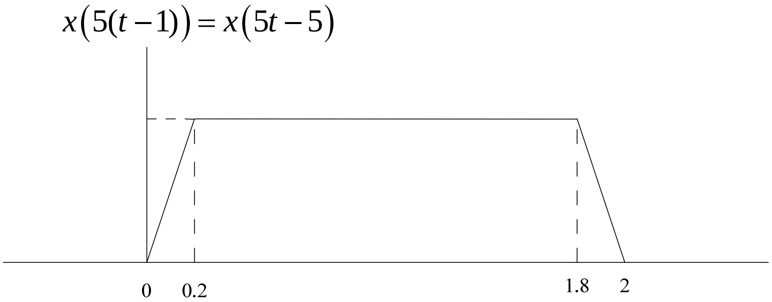
2.

(a) (10%)





(b) (10%)

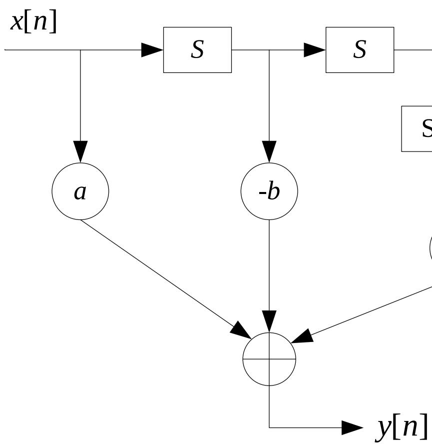


3. (15%)

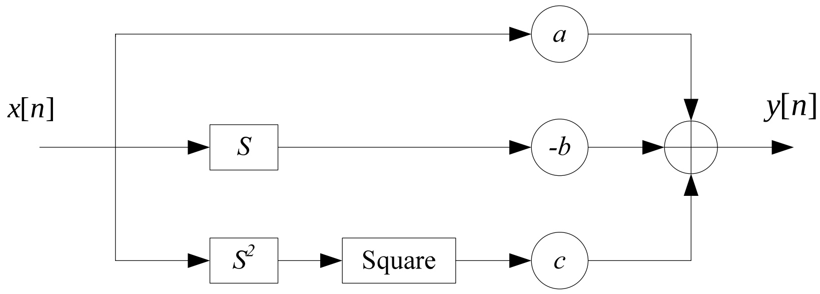
$$\begin{aligned} y(t) &= y_1(t) + y_2(t) - y_4(t) \\ &= x_1(t)x_1(t-1) + |x_2(t)| - \cos(1+2x_3(t)) \\ &= x(t)x(t-1) + |x(t)| - \cos(1+2x(t)) \end{aligned}$$

4. $y[n] = ax[n] - bx[n-1] + cx^2[n-2] = \left(a - bS + cS^2\right)\{x[n]\}$

(10%) Cascade implementation of operator H:



(10%) Parallel implementation of operator H:



5. (25%)

	Memory-less	Stable	Causal	Linear	Time Invariant
$y(t)=\cos(x(t))$	O	O	O	×	O
$y[n]=2x[n]u[n]$	O	O	O	O	×
$y[n]=\log_{10}(\lvert x[n]\rvert)$	O	×	O	×	O
$y(t)=\int_{-\infty}^{t/2} x(\tau)d\tau$	×	×	×	O	×
$y[n]=\sum_{k=-\infty}^{n} x[k+2]$	×	×	×	O	O

(1)

$$y_1(t)=\cos(\alpha x_1(t));\ y_2(t)=\cos(\beta x_2(t))$$

$$y_3(t)=\cos(\alpha x_1(t)+\beta x_2(t))$$

$$\neq \cos(\alpha x_1(t))+\cos(\beta x_2(t))=y_1(t)+y_2(t) \quad \Rightarrow \text{nonlinear}$$

(2)

$$y_1[n-n_0]=2x_1[n-n_0]u[n-n_0]$$

$$x_2[n]=x_1[n-n_0]$$

$$y_2[n]=2x_2[n]u[n]=2x_1[n-n_0]u[n]\neq y_1[n-n_0] \quad \Rightarrow \text{time-varying}$$

(3)

- $x[n]=0,\ |y[n]|=|\log_{10}(0)|=\infty\ \Rightarrow$ unstable
- $y_1[n]=\log_{10}(|\alpha x_1[n]|);\ y_2[n]=\log_{10}(|\beta x_2[n]|)$

  $y_3[n]=\log_{10}(|\alpha x_1[n]+\beta x_2[n]|)\neq y_1[n]+y_2[n]\quad\Rightarrow$ nonlinear

(4)

- Since the integrated range starts from negative infinite, the system has memory.
- $\left|y(t)\right|=\left|\int_{-\infty}^{t/2}x(\tau)d\tau\right|\leq\int_{-\infty}^{t/2}\left|x(\tau)\right|d\tau=M_x\int_{-\infty}^{t/2}1d\tau=M_x\left(\frac{t}{2}+\infty\right)=\infty\Rightarrow$ unstable
- If $t<0$, then y(t) is noncausal due to $t<0.5t$.
- $y_1(t-t_0)=\int_{-\infty}^{(t-t_0)/2}x_1(\tau)d\tau$

  $x_2(t)=x_1(t-t_0)$

  $y_2(t)=\int_{-\infty}^{t/2}x_2(\tau)d\tau=\int_{-\infty}^{t/2}x_1(\tau-t_0)d\tau$

  $\overset{\tau'=\tau-t_0}{=}\int_{-\infty}^{t/2-t_0}x_1(\tau')d\tau'\neq y_1(t-t_0)\quad\Rightarrow$ time-varying

(5)

- $y[n]=\sum_{k=-\infty}^{n}x[k+2]=\cdots+x[n]+x[n+1]+x[n+2]\ \Rightarrow$ memory and

  noncausal
- $\left|y[n]\right|=\left|\sum_{k=-\infty}^{n}x[k+2]\right|\leq\sum_{k=-\infty}^{n}\left|x[k+2]\right|\leq(n+1+\infty)\cdot M_x=\infty\ \Rightarrow$ unstable