Homework No. 1
Due 18:10, October 7, 2010

1. Determine whether the following signals are periodic, and for those which are, find the fundamental period: (20%)
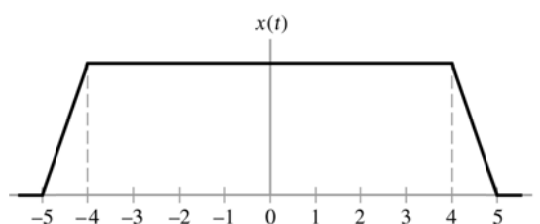
(1) $x[n] = \sin\left(\dfrac{6\pi}{7}n + 1\right)$

(2) $x(t) = \left[\cos\left(2t - \dfrac{\pi}{3}\right)\right]^2$

(3) $x(t) = \sum_{k=-\infty}^{\infty}(-1)^k\,\delta(t - 2k)$

(4) $x[n] = \cos\left(\dfrac{\pi}{2}n\right)\cos\left(\dfrac{\pi}{4}n\right)$

2.

    (1)   The trapezoidal pulse $x(t)$ of Fig. 2 is time scaled, producing the equation $y(t) = x(at)$. Sketch $y(t)$ for $a = 20$ and 0.1. (10%)

Figure 2

    (2) Sketch the trapezoidal pulse $y(t)$ related to that of Fig. 2 as follows

$$y(t) = x\big(5(t - 1)\big) \quad (10\%)$$

3. A system consists of several subsystems connected as shown in Fig. 1. Express $y(t)$ as a function of $x(t)$. (15%)

$$H_1 : y_1(t) = x_1(t)x_1(t - 1);$$
$$H_2 : y_2(t) = \left|x_2(t)\right|;$$
$$H_3 : y_3(t) = 1 + 2x_3(t);$$
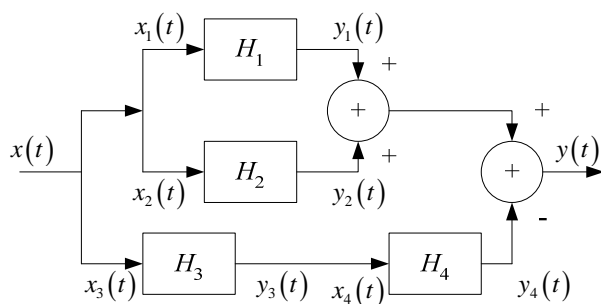$$H_4 : y_4(t) = \cos\big(x_4(t)\big).$$

Figure 1

4. The output of a discrete-time system is related to its input $x[n]$ as follows:

$$y[n] = a \cdot x[n] - b \cdot x[n-1] + c \cdot x^2[n-2]$$

Let the operator S^k denote a system that shifts the input $x[n]$ by k time units to produce $x[n-k]$. Draw the block diagrams representation for this system by using (a) cascade implementation and (b) parallel implementation. (20%)

5. The system that follow have input $x(t)$ or $x[n]$ and output $y(t)$ or $y[n]$. For each system, determine whether it is (i) memoryless, (ii) stable, (iii) causal, (iv) linear, and (v) time invariant. (25%)

(1) $y(t) = \cos(x(t))$; (2) $y[n] = 2x[n]u[n]$; (3) $y[n] = \log_{10}(|x[n]|)$;

(4) $y(t) = \int_{-\infty}^{t/2} x(\tau)d\tau$; (5) $y[n] = \sum_{k=-\infty}^{n} x[k+2]$.