Homework No. 8

Due 15:10, January 6, 2010

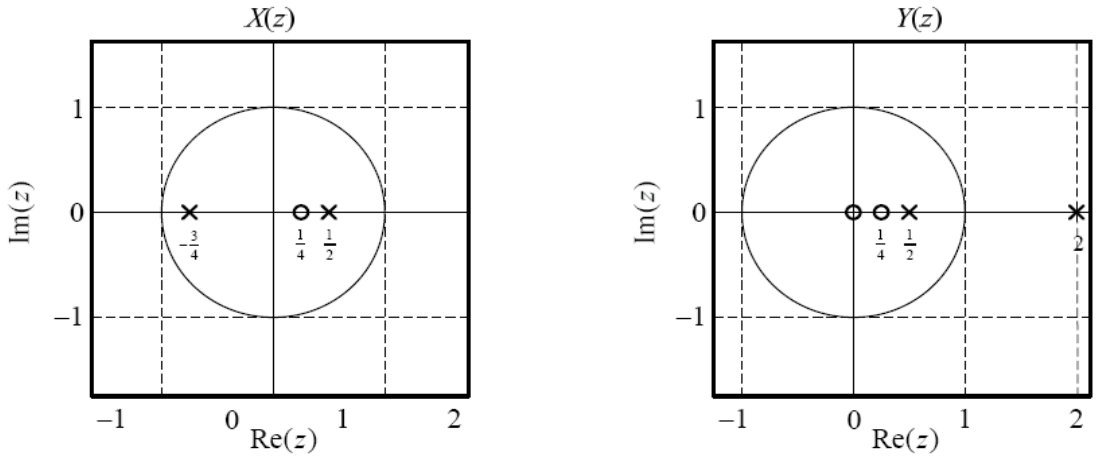
1. Using any method, determine the inverse z-transform for each of the following:

   (1) $X(z)=\dfrac{1}{\left(1+0.5z^{-1}\right)^{2}\left(1-2z^{-1}\right)\left(1-3z^{-1}\right)}$, stable sequence. (10%)

   (2) $X(z)=e^{z^{-1}}$. (10%)

   (3) $X(z)=\dfrac{z^{3}-2z}{z-2}$, left-sided sequence. (10%)

2. The signal $y[n]$ is the output of an LTI system with impulse response $h[n]$ for a given input $x[n]$. Throughout the problem, assume that x[n] is stable and has a z-transform $X(z)$ with the pole-zero diagram shown in the following figure. Also assume that $y[n]$ is stable and has a z-transform $Y(z)$ with the pole-zero diagram shown in the following figure, too.



   (1) What is the ROC of $Y(z)$? (5%)
   (2) Is $y[n]$ left-sided, right-sided, or two-sided? (5%)
   (3) What is the ROC of $X(z)$. (5%)
   (4) Is $x[n]$ a causal sequence? (5%)
   (5) What is $x[0]$? (5%)
   (6) Draw the pole-zero diagram of $H(z)$, and specify its ROC. (5%)
   (7) Is $h[n]$ anticausal? (5%)

3. We are given the following five facts about a discrete-time signal $x[n]$ with z-transform $X(z)$:
   (1) $x[n]$ is real and right-sided.
   (2) $X(z)$ has exactly two poles.
   (3) $X(z)$ has a pole at $z = 0.5e^{j\pi/3}$.
   (4) $X(z)$ has a two zeros at the origin.
   (5) $X(1) = 8/3$.
   Determine $X(z)$ and specify its ROC. (15%)

4. A causal LTI system is describe by the difference equation
$$y[n] = y[n-1] + y[n-2] + x[n-1].$$
   (1) Find the system function $H(z) = Y(z)/X(z)$ for this system and specify its ROC. (10%)
   (2) Find a stable (noncausal) unit impulse response that satisfies the difference equation. (10%)