Homework No. 7

Due 15:10, Dec. 30, 2009

1. Consider the bilateral Laplace transform given by $X(s)=\frac{2s}{s^2-4}$.
   (1) Determine the corresponding stable time-domain signal $x(t)$. (10%)
   (2) Determine the corresponding causal time-domain signal $x(t)$. (10%)
   (3) Determine the corresponding anti-causal time-domain signal $x(t)$. (10%)
   (4) Explain why it is impossible to find a causal and stable solution. (10%)

2. Given the unilateral Laplace transform pair $\cos(2t)u(t)\xleftrightarrow{\mathcal{L}_u} X(s)$, determine the time-domain signals corresponding the following Laplace transforms:
   (1) $(s+1)X(s)$ (5%)
   (2) $X(s+2)$ (5%)
   (3) $s^{-2}X(s)$ (5%)
   (4) $\frac{d}{ds}\left(e^{-3s}X(s)\right)$ (5%)

3. We are given the following five facts about a real signal $x(t)$ with Laplace transform $X(s)$:
   (1) $X(s)$ has exactly two poles.
   (2) $X(s)$ has no zeros in the finite s-plane.
   (3) $X(s)$ has a pole at $s = -1+j$.
   (4) $e^{2t}x(t)$ is not absolutely integrable.
   (5) $X(0)=8$.
   Determine $X(s)$ and specify its ROC. (20%)

4. Consider a continuous-time LTI system for which the input $x(t)$ and output $y(t)$ are related by the differential equation
$$\frac{d^2y(t)}{dt^2}-\frac{dy(t)}{dt}-2y(t)=\frac{dx(t)}{dt}-x(t).$$
   Let $H(s)$ denote the Laplace transform of $h(t)$, the system impulse response .
   (1) Determine $H(s)$ as a ratio of two polynomials in s and plot the pole-zero diagram of $H(s)$. (10%)
   (2) Draw the direct-form-II of $H(s)$. (10%)