

Homework No. 6
Due 18:10, May 14, 2009

1. You are given $x[n] = n(1/2)^{|n|} \xleftrightarrow{DTFT} X(\Omega)$. Without evaluating $X(\Omega)$, find $y[n]$ if

(1) $Y(\Omega) = \text{Re}\{X(\Omega)\}$ (6%) (2) $Y(\Omega) = \frac{d}{d\Omega} X(\Omega)$ (6%)

(3) $Y(\Omega) = X(\Omega) \otimes X(\Omega - \pi/2)$ (6%) (4) $Y(\Omega) = X(\Omega) + X(-\Omega)$ (6%)

(5) $Y(\Omega) = e^{-4j\Omega} X(\Omega)$ (6%)

2. Consider the two sequences: $x_1[n] = u[n] - u[n-8]$ and $x_2[n] = u[n] - u[n-4]$.

Let the 8-point DFT's of these two sequences be multiplied, and then let $y[n]$ be the inverse DFT of the result.

- (1) Compute and sketch $y[n]$. (15%)
- (2) Specify which points in $y[n]$ are corresponding to the points that would be obtained in a linear convolution of $x_1[n]$ and $x_2[n]$. (10%)
- (3) How to form two sequences $\hat{x}_1[n]$ and $\hat{x}_2[n]$ from $x_1[n]$ and $x_2[n]$ respectively such that the linear convolution of $x_1[n]$ and $x_2[n]$ can be computed via the circular convolution of $\hat{x}_1[n]$ and $\hat{x}_2[n]$. (10%)

3. Let $x[n]$ and $h[n]$ be the signals with the following Fourier transforms:

$$X(\Omega) = 3e^{j\Omega} + 1 - e^{-j\Omega} + 2e^{-j3\Omega}$$

$$H(\Omega) = -e^{-j\Omega} + 2e^{-j2\Omega} + e^{j4\Omega}$$

Determine $y[n] = x[n] * h[n]$. (15%)

4. Use the tables of transforms and properties to find the DTFTs or the inverse DTFTs of the following signals: (20%)

(1) $x[n] = (n-2)(u[n+4] - u[n-5])$ (2) $x[n] = \left(\frac{1}{3}\right)^n u[n+2]$

(3) $X(\Omega) = \left[e^{-j2\Omega} \frac{\sin(15\Omega/2)}{\sin(\Omega/2)} \right] \otimes \left[\frac{\sin(7\Omega/2)}{\sin(\Omega/2)} \right]$ (4) $X(\Omega) = \cos(4\Omega) \left[\frac{\sin(3\Omega/2)}{\sin(\Omega/2)} \right]$