

Homework No. 5

Due 18:10, Nov. 26, 2009

1. Let $x[n]$ be a periodic signal with period N and Fourier coefficients a_k .
 - (1) Express the Fourier coefficients b_k of $|x[n]|^2$ in terms of a_k . (10%)
 - (2) If the coefficients a_k are real, is it guaranteed that the coefficients b_k are also real? (10%)

2. When the impulse train $x[n] = \sum_{k=-\infty}^{\infty} \delta[n-4k]$ is the input to a particular LTI system with frequency response $H(e^{j\Omega})$, the output of the system is found to be $y[n] = \cos\left(\frac{5\pi}{2}n + \frac{\pi}{4}\right)$. Determine the values of $H(e^{jk\pi/2})$ for $k = 0, 1, 2$, and 3. (20%)

3. You are given $x[n] = n(1/2)^{|n|} \xleftrightarrow{DFT} X(\Omega)$. Without evaluating $X(\Omega)$, find $y[n]$ if
 - (1) $Y(\Omega) = \text{Re}\{X(\Omega)\}$ (5%)
 - (2) $Y(\Omega) = dX(\Omega)/d\Omega$ (5%)
 - (3) $Y(\Omega) = X(\Omega) + X(-\Omega)$ (5%)
 - (4) $Y(\Omega) = e^{-4j\Omega}X(\Omega)$ (5%)

4. Let $x[n]$ and $h[n]$ be the signals with the following Fourier transforms:

$$X(e^{j\Omega}) = 3e^{-j\Omega} + 1 - e^{j\Omega} + 2e^{j3\Omega}$$

$$H(e^{j\Omega}) = 2e^{-j2\Omega} - e^{-j\Omega} + e^{j4\Omega}$$

Determine $y[n] = x[n] * h[n]$. (15%)

5. Consider the finite-length sequence $x[n] = 2\delta[n] + \delta[n-1] + \delta[n-3]$.
 - (1) Compute the five-point DFT $X[k]$. (10%)
 - (2) If $Y[k] = X^2[k]$, determine the sequence $y[n]$ with five-point inverse DFT for $n = 0 \sim 4$. (10%)
 - (3) If N -point DFTs are used here, how should we choose N such that $y[n] = x[n] * x[n]$, for $0 \leq n \leq N-1$. (5%)