Homework 8

Due 16:20, June 12, 2008

1. Find the Laplace transform of following signals; indicate the ROC of each signal with figure also. (15%)

   (a) $x(t) = -e^{-at}u(t)$.

   (b) $x(t) = e^{-2t}u(t) + e^{-t}(\cos 3t)u(t)$

   (c) $x(t) = \delta(t) - \frac{4}{3}e^{-t}u(t) + \frac{1}{3}e^{2t}u(t)$

2. Consider two right-sided signals x(t) and y(t) related through the differential equations

$$\frac{dx(t)}{dt} = -2y(t) + \delta(t)$$

   and

$$\frac{dy(t)}{dt} = 2x(t)$$

   Determine Y(s) and X(s), along with their regions of convergence. (15%)

3. The system function of a causal LTI system is

$$H(s) = \frac{s+1}{s^2+2s+2}$$

   Determine and sketch the response y(t) when the input is

$$x(t) = e^{-|t|},\ -\infty < t < \infty \ . \ (15\%)$$

4.
   (I) A system has the indicated transfer function $H(s)$. Determine the impulse response, assuming (a) that the system is causal and (b) that the system is stable. (10%)

$$H(s) = \frac{s^2+2s+2}{s^2-1}$$

   (II) A stable system has the indicated input x(t)and output y(t). Use Laplace transforms to determine the transfer function and impulse response of the system. (10%)

$$x(t) = e^{-2t}u(t)\ , \quad y(t) = -2e^{-t}u(t) + 2e^{-3t}u(t)$$

5. Draw a direct form representation for the causal LTI systems with the following system functions. (10%)

(a) $H(s) = \dfrac{s^2 - 5s + 6}{s^2 + 7s + 10}$

(b) $H(s) = \dfrac{s}{(s+2)^2}$

6. The signal

$$y(t) = e^{-2t}u(t)$$

is the output of a causal all pass system for which the system function is

$$H(s) = \frac{s-1}{s+1}$$

(a) Find and sketch at least two possible inputs x(t) that could produce y(t).

(b) What is the input x(t) if it is known that

$$\int_{-\infty}^{\infty} |x(t)|\, dt < \infty\ ?$$

(c) What is the input x(t) if it is known that a stable (but not necessarily causal) system exists that will have x(t) as an output if y(t) is the input? Find the impulse response of this filter, and show by direct convolution that it has the property claimed (i.e., that is $x(t) * h(t) = y(t)$)

(25%)